Journal of Mines, Metals and Fuels, *71(10): 1645-1656; 2023. DOI: 10.18311/jmmf/2023/35867*

Print ISSN : 0022-2755

Journal of Mines, Metals and Fuels

Contents available at: *www.informaticsjournals.com/index.php/jmmf*

Free Vibration Response of Functionally Graded Porous Metallic Plates Embedded with Piezoelectric Layers

Prashant Kumar Choudhary[1], Ranjan Kumar[2] and Subodh Kumar[3]*

[1]*Department of Mechanical Engineering, GMR Institute of Technology, Razam - 502329, Andhra Pradesh, India*
[2]*Department of Mechanical Engineering, Swami Vivekananda University, Kolkata - 700121, West Bengal, India*
[3]*Department of Mechanical Engineering, SR University, Warangal - 517326, Telangana, India; subodhkumarism@gmail.com*

Abstract

The objective of this study is to determine the natural frequencies of Functionally Graded (FG) metallic plates comprising piezoelectric layers on both the top and bottom surfaces. The material characteristics of the FG plates are expected to exhibit a gradual variation along the thickness direction in accordance with a power-law model along with porosity. The governing equations are derived using the principle of virtual displacements, taking into account the first-order shear deformation plate theory. A commercial finite element programme in ANSYS Parametric Design Language (APDL) is developed to compute the natural frequency and mode shapes of functionally graded porous plates embedded with piezoelectric layers. The obtained natural frequencies results are used for different boundary condition to show their variations with respect to the constituent volume fractions, boundary condition, and piezoelectric thickness for the parametric study. The present paper highlights some important characteristics of Functionally Graded Materials (FGM) plate embedded with piezoelectric layers that can be advantageous in the design of smart structures.

Keywords: *Free Vibration, Functionally Graded Material (FGM) Plate,Piezoelectric Layer, Power Law Distribution*

1.0 Introduction

Smart structures have drawn attention in various field of engineering like aerospace, civil, mechanical and even bio-medical engineering. Typically, a smart structure consists of a host structure that has sensors and actuators integrated into it. These smart structures could be useful for a number of engineering applications, including shape control, vibration, noise reduction, and structural health monitoring. One category of smart or intelligent structures is piezoelectric coupled structures. These coupled piezoelectric material possesses an electromechanical property that has both a direct and indirect piezoelectric effect[1]. According to the direct piezoelectric effect, the piezoelectric sensor can respond to vibrations by generating a voltage. A feedback gain processes and amplifies this voltage before applying it to an actuator. Due to the inverse piezoelectric effect, the actuator then generates a control force. Structure vibration may be adequately suppressed if the control force is appropriate.

The frequency response of functionally graded porous plates combined with piezoelectric layers is developed in this study. Functionally Graded Materials (FGMs) have spatial changes in material properties that can be used to optimise structural component performance. Porous materials, on the other hand, have a porous microstructure that provides additional advantages such as lightweight and improved energy absorption. The incorporation of piezoelectric layers enables active

**Author for correspondence*

control and sensing. Piezoelectric materials are coupled with FGM Plates for broader application in different field of engineering. These FGMs are an unique class of sophisticated composite materials with smooth constituent elements and continuously varied in a desired direction and in a predetermined manner. FGM plates are composed of two distinct isotropic constituent phases known as ceramic and metallic constituents. The combination of piezoelectric materials with FGM plates has various advantages and opens up new avenues for technical applications. the combination of piezoelectric materials and FGM plates offers numerous benefits, including active vibration control, energy harvesting, sensing and structural health monitoring, shape control and actuation, acoustic wave generation and control, multi-functionality, and lightweight design. These benefits make the combination of piezoelectric materials and FGM plates a potential strategy for a wide range of technical applications, including aerospace and robotics, as well as energy harvesting and structural health monitoring systems.

2.0 Literature Pertaining to Prior Work

The studies on the FGM plates having piezoelectric layers incorporated is limited. Thus, few importature literatures pertaining to accurate free vibration of FGM and piezoelectric layes based FGM plates is discussed to show the background of this current work. Thai *et al.*[2], conducted studied on the free vibration analysis of FG plates using a First-Order Shear Deformation Theory (FSDT)[2]. Tu *et al.*[3] investigated the vibration analysis of functionally graded plates in thermal environments. They employed an unknown Higher-Order Shear Deformation Theory (HSDT) for their analysis[3]. Abrate[4] conducted investigation on various aspects of functionally graded plates, including free vibrations, buckling, and static deflections. The author used different models in their research[4].

Kumar and Jana[5], examined the accurate frequency behaviour of a stepped Functionally Graded Material (FGM) plate. Their investigation used the dynamic stiffness method[5]. Kumar and his co-authors have extensively researched accurate frequency prediction techniques for various types of FGM plates, including thin uniform, stepped, and porous FGM plates. They employed the dynamic stiffness methodology combined with the Wittrick-Williams algorithm for their analyses[6-9]. Zhaoyang Hu[10] investigated the free vibration behaviour of non-levy FG plates using a novel analytical method known as the symplectic superposition method[10]. Mohamadreza Jafarinezhad *et al.*[11], explored the free vibration behaviour of FG annular plates. They employed analytical methods for their investigation[11]. Narayanan N. I. and Banerjee S.[12] determined the free vibration characteristics of FG plates, both with and without circular cutouts. The author used the Finite Element Method (FEM)[12]. Jana K. *et al.*[13], the authors focused on the free dynamic analysis of a rectangular FG plate[13]. Sah S.K. and Ghosh A.[14] studied the Dynamic and stability analysis of Functionally Graded Material (FGM) plates with the finite element method in their research[14]. Parida S. and Mohanty S.C.[15] investigated the free vibration response of a functionally graded rotating plate within a thermal environment[15]. Tran, M.T. and Thai [16] conducted research on the dynamic behavior of multi-directional FG plates with variable thickness[16]. Huang X. L. and Shen H. S.[17] provided an analytical solution for the nonlinear dynamic analysis of Functionally Graded Material (FGM) plates integrated with piezoelectric layers under thermal loading condition[17]. Liu G. R. *et al.*, studied the active vibration control of plates composed of Functionally Graded Piezoelectric Material (FGPM) integrated with actuator[18]. Talha M. and Singh B. developed the static and free vibration analyses of Functionally Graded Material (FGM) plates using with Higher-Order Shear Deformation Theory (HSDT)[19] and this problem is solved by the FEM.

During fabrication of FGM plates, porosities and defects are the main issues that cannot be avoided. To this purpose, it is important to assume that the effect of porosity during the structural analysis. The porosity effects have been investigated in a number of studies. As an example, Rezaei A.S. *et al.* conducted a free vibration behaviour of FGM plates with porosities and explored various geometrical and material characteristics on the natural frequency of the plates[20]. Wang *et al.* investigated the vibration analysis of longitudinally moving functionally graded material rectangular plates with porosities[21]. In a thermal environment, Wang and Zu (2017) conducted research to study the vibration characteristics of FGM porous plates[22]. Shahsavari D *et al.* studied a novel quasi-3D theory to analyze the free vibration of FG porous plates[23]. Bansal *et al.* used FEM and Navier's solution to investigate the vibration characteristics of Functionally Graded Porous (FGP) plates, specifically considering partially supported Boundary Conditions (BCs)[24].

Based on the literature review, it is evident that there is a limited amount of research conducted using the FEM to analyze the free vibration of FGM porous plates integrated with piezoelectric layers. Hence, the primary objective of this study is to develop a FE model for conducting fequency analysis of FG porous plates incorporating piezoelectric layers. In this work, the free vibration response of FGM plate integrated with piezoelectric layers with and without porosity. The FE model is based on FSDT and principal of virtual displacements. A Matlab program has been made using the derived formulation. The results is validated with some published literature and finite element software ANSYS. The primary objective of the research is to assess how functional grading effects the mechanical behaviour of the plate. Besides that, the natural frequency response along with different mode shapes of the FGM plates is investigated for different boundary condition and discussed in detail below in the results and discussion section.

3.0 Functionally Graded Materials (FGM) Plates

Figure 1 depicts a plate structure consisting of functionally graded materials integrated with a piezoelectric layer that act as both an actuator and a sensor layer. The piezoelectric actuator layer is on top in this plate, while the piezoelectric sensor layer is on the bottom. The intermediate layer is made up of a mix of ceramic and metal components.

Table 1 shows the FGM plates' and PZT-4 material properties. The PZT-5A piezoelectric material characteristics are: E=63 GPa, ν =0.3, ρ =7600 kg/m^3, d_{31} =

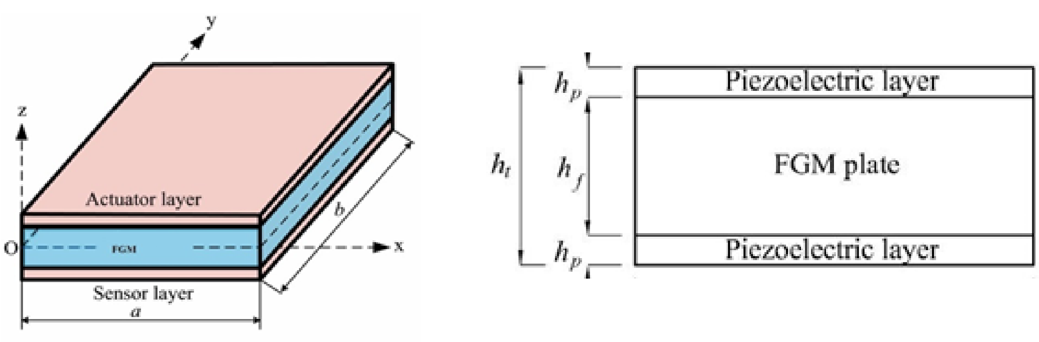


Figure 1. FGM Plate Schematic Diagram with Piezoelectric Layer.

Table 1. Material properties of FGM plate and Piezoelectric layers

Elastic Property	Core plate		Piezoelectric layers
	Aluminum oxide	Ti-6A1-4V	PZT-4
E GPa	320.24	105.70	
ν	0.26	0.26	
C_{11} GPa			132
C_{12} GPa			71
C_{33} GPa			115
C_{13} GPa			73
C_{55} GPa			26
e_{31} (C/m^{-2})			-4.1
e_{33} (C/m^{-2})			14.1
e_{15} (C/m^{-2})			10.5
ρ kg/m^3	3750	4429	7500

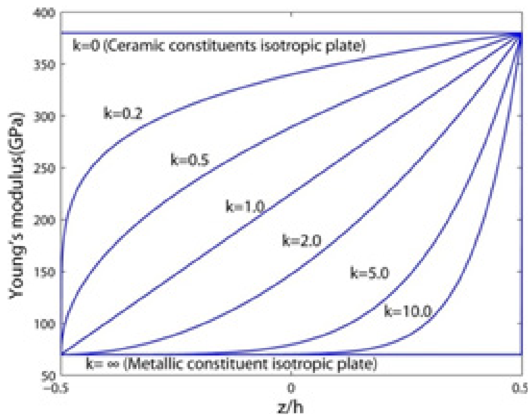


Figure 2. Plot of the effective Young's modulus varition along the thickness of FGM plates

254×10^{-12} m/V and $d_{32} = 254\times10^{-12}$ m/V, $k_{33} = 15\times10^{-9}$ F/m. The power-law model is taken as the basis for describing the variation in material characteristics along thickness directions. This is the most widely adopted model. The rule of mixture model is provided by:

$$P(Z) = (P_C - P_m)\left(\frac{Z}{h} + \frac{1}{2}\right)^k + P_m \quad (1)$$

P(Z) denotes the FGM plate's effective material Characteristics, such as density and Young's Modulus (E(Z)). The FGM plate is made up ceramic and metallic components and are indicated by the subscripts C and m in the material constituents. The essential parameter "*k*" with a value ranging from 0 to infinity represents the power-law index. In the thickness direction of a FGM plate, Figure 2 shows how the Young's modulus varies for various values of "*k*."

3.1 Functionally Graded Porous Materials Plate

A functionally graded porous materials plate is considered with materials properties variation in along thickness directions as shown in Figure 3. The power-law model is considered. This is the most widely adopted model. According to this model, the rule of mixture, which is provided by:

$$P(Z) = \left((P_C - P_m)\left(\frac{Z}{h} + \frac{1}{2}\right)^k + P_m\right)(1 - \emptyset(Z)) \quad (2)$$

$$V_c(z) = \left(\frac{z}{h} + 0.5\right)^k$$

Where $\emptyset(Z) = \varphi cos\left(\frac{\pi Z}{h}\right)$ and $V_c(z)$ is the volume fraction of

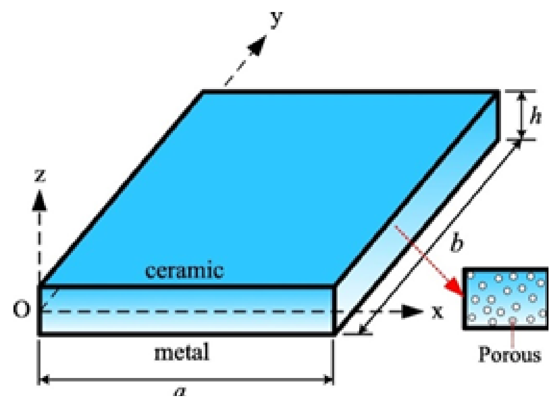


Figure 3. Schematic diagram of FGM Plate with porosity.

P(Z) denotes the FGM plate's effective material Characteristics, such as density and Young's Modulus (E(Z)). The material's properties subscript c and m represent pure ceramic and metallic constituents, respectively. h represent thickness of functionally graded porous materials plate's. Z is the thickness coordinate in the z-axis. Figure 3 depicts the geometry of a functionally graded materials plate embedded with a porosity distribution. The porosity distribution factor is given by $\emptyset(Z)$. The φ range is between 0 and 0.5 (Kim J. *et al.* 2019)[25]. Figure 4 depicts the effective Youngs

Table 2. Mechanical characteristics of Si_3N_4/SUS304.

Materials properties	*E* (GPa)	Densities ρ (Kg/m^3)	ν
Si_3N_4 (ceramic)	348.46	2370	0.3
SUS304 (metal)	201.04	8166	0.3

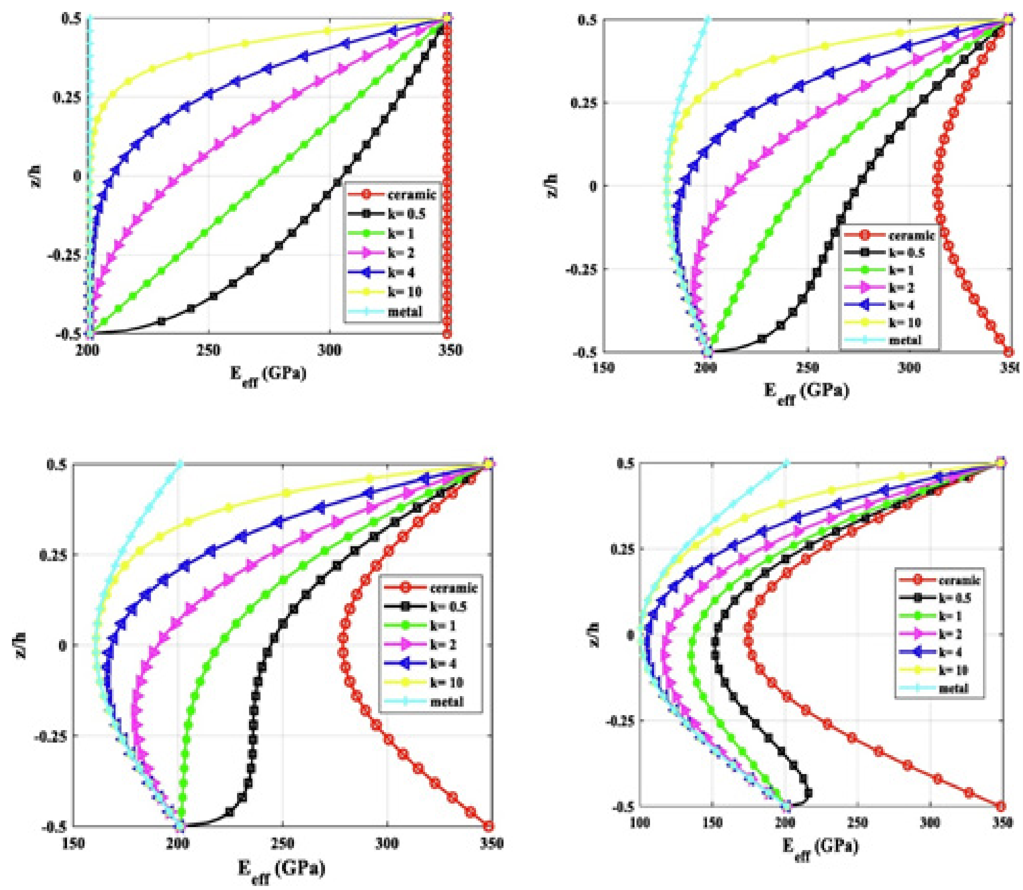


Figure 4. Variation in the power-law index k and the effective elastic modulus for different value of porosity (φ =0,0.1,0.2,0.5)

modulus across thickness for various values of k and porosity. Table 2 displays the material parameters of the constituent materials (Si_3N_4/SUS_30_4).

4.0 Finite Element Formulation

4.1 Displacement and Strains

“Based on the First-Order Shear Deformation Theory (FSDT), the displacement field is considered as :

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\emptyset_x(x,y)\\
v(x,y,z) &= v_0(x,y) + z\emptyset_y(x,y)\\
w(x,y,z) &= w(x,y)
\end{aligned}
\tag{3}
$$

Where u_0, v_0, w are the displacement components at the midplane of the plate in the x,y,z direction respectively.

According to the FSDT, the strain components connected to changes in curvature and twist:

$$
\begin{aligned}
\varepsilon_{11} &= \frac{\partial u_o}{\partial x} + z\frac{\partial \phi_1}{\partial x}\\
\varepsilon_{22} &= \frac{\partial v_o}{\partial y} + z\frac{\partial \phi_2}{\partial y}\\
\varepsilon_{12} &= \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} + z\left(\frac{\partial \phi_1}{\partial y} + \frac{\partial \phi_2}{\partial x}\right)\\
\varepsilon_{13} &= \frac{\partial w}{\partial x} + \phi_1\\
\varepsilon_{23} &= \frac{\partial w}{\partial y} + \phi_2
\end{aligned}
\tag{4}
$$

The expression of electric field in terms of potential (β) :

$$E_x = -\frac{\partial \beta}{\partial x} \qquad E_y = -\frac{\partial \beta}{\partial y} \qquad E_z = -\frac{\partial \beta}{\partial z} \tag{5}$$

Constitutive relation:
The linear constituent connections of piezoelectric materials define their electrical and mechanical interactions:

$$\{\sigma\} = [Q]\{\varepsilon\} - [e]\{E\}$$
$$\{D\} = [e]^T\{\varepsilon\} + [\mathrm{n}]\{E\} \tag{6}$$

Where σ is stress vector, ε is strain vector, $D = \{D_x, D_y\,D_z\}^T$ is electric displacement, $E = \{E_x, E_y\,E_z\}^T$ is electric field, [e] is the piezoelectric coupling strain constants matrix, [Q], are matrices of elastic constant, [n] is dielectric constants..

The total potential energy U can be calculated as:

$$U = \frac{1}{2}\left[\int_V \{\varepsilon\}^T\{\sigma\}\right]dV + \frac{1}{2}\left[\int_V \{\varepsilon_s\}^T\{\tau\} - \int_V \{E\}\{D\}\right]dV \tag{7}$$

$$\{\varepsilon\} = [B_u]\{u^e\} \tag{8}$$

The electric field (E) is represented by nodal variables as:

$$\{E\} = -\nabla\beta = -\{\beta^e\}[B_\beta] \tag{9}$$

Where ∇ is gradient operator.

$$[B_\beta] = \nabla[N_\beta]$$

The equation of motion can be expressed as:

$$[M_{uu}]\{\ddot{u}\} + [K_{uu}]\{u\} + [K_{u\beta}]\{\beta\} = [F_m] \tag{10}$$

The eigenvalue equation are:

$$\left|\left([K] - \omega^2[M_{uu}]\right)\right| = 0 \tag{11}$$

Where ω is the natural frequency, K is global stiffness and M_{uu} is the global mass matrices.

5.0 Finite Element Modeling in ANSYS

The FGM Plate has been modeled in ANSYS Parametric Design Language (*APDL v18.1*) using solid *SOLID 186* elements. Piezoelectric layers are modeled by SOLID 5 coupled field elements. The SOLID5 element having a 8 nodes, each with four degrees of freedom (*ux, uy, uz, and volt*). Figure 5 depicts the mesh model of FGM plate coupled with piezoelectric layers.

6.0 Anaysis of Results

In this section, free vibraion results are described for the functionally graded materials plate interated with piezoelectric layers for different constituent materials with and without porosity consideration.

6.1 Functionally Graded Plate (FGM)

This current section is used to validate present result with published result. For this, it is considered that the free vibraton response of FGM plate for simply supported

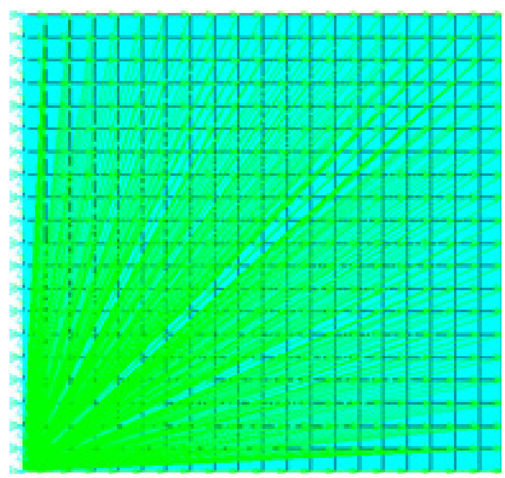


Figure 5. Meshed model of FGM Plate oupled withPiezoelectric Layers in ANSYS.

(SSSS) edge condition and compared the results with the result of author[26], for various power law index k's . The geometrical dimension and material characteristics is considered same as in reference. The side and thickness of square FGM plate are 0.2 *m* and 0.025 *m*. For materials properties, silicon nitride and stainless steel are chosen to be the FGM plate's component materials. The comparison of natural frequencies with published literature[26] is presented in Table 3. From the table, it is seen that present results are close to 0.2% error with published results.

Table 3. Non-dimensional natural frequency comparison for Si_3N_4/SUS_30_4 square FGM plates with published literature[26].

k	Present study	Huang XL., and Shen HS.[26]
Silicon nitride	12.52	12.49
k =0.5	8.66	8.67
k =1	7.57	7.55
k =2	6.83	6.77
Stainless steel	5.42	5.40

6.2 FGM Plate with Piezoelectric Layers

In this section, we will study the free vibration analysis of FGM plate with surface bonded on the top and the bottom with piezoelectric layers. The Functionally Graded Materials (FGM) plate is composed of Al and Al_2O_3 integrated with PZT-4 piezoelectric layers on top and bottom. The thickness of piezoelectric layers is 0.1h. The material properties of the Al, Al_2O_3 and PZT-4 are the same as given in literature[27]. The results are reported for various values of values (k = 0,0.5,1,2) and different boundary conditions SSSS, SCSC in Table 4 and Table 5 respectively.

In another case, the natural frequency of a FGM plate with surface bonded piezoelectric layers PZT-5A on the top and bottom is determined. The core plate and PZT-5A have 400 and 400 mm dimensions, respectively. The core Plate and PZT-5A have thicknesses of 5 mm and 0.5 mm, respectively.

By changing the volume fraction of ceramic constituent, the influence of constituent volume fraction is examined. This can be done by changing the power law index k 's value. For instance, k = 0 suggests that the FGM plate solely contains Ti-6A1-4v. Figure 3 illustrates how the volume fraction of ceramic constituent Ti-6A1-4v

Table 4. Natural frequency (Hz) of the SSSS FGM plates integrated with piezoelectric layers

	Natural frequency (Hz)	
	Present Method	**Farsangi & Saidi (2012)**
0	432.15	433.74
0.5	376.66	377.93
1	348.94	350.09
2	327.53	328.72

Table 5. Natural frequency (Hz) of the SCSC FGM plates integrated with piezoelectric layers

	Natural frequency (Hz)	
	Present Method	**Farsangi & Saidi (2012)**
0	622.20	626.40
0.5	541.74	546.20
1	501.00	505.92
2	468.71	474.48

Table 6. Natural frequency (Hz) for the actuator layers that are bonded to the top and bottom surfaces of the cantilevered FGM plates

Mode no	k = 0	k = 0.2	k = 0.5	k = 1	k = 15	k = 100
1	25.35	29.50	32.45	36.98	43.43	45.85
2	62.74	73.35	81.10	87.40	109.01	114.80
3	157.10	183.75	202.45	217.85	271.12	287.30
4	200.14	233.64	257.12	276.42	344.35	364.65
5	228.13	267.32	294.50	316.85	395.84	419.05

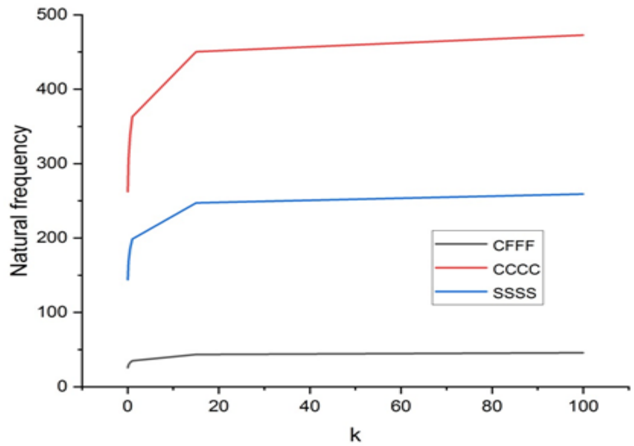


Figure 6. Comparison of natural frequencies of cantilevered, fully clamped and simply supported FGM plates integrated with piezoelectric layers for various value of k.

decreases as k rises. The FGM plate is almost entirely made of aluminium oxide when k goes towards 1.

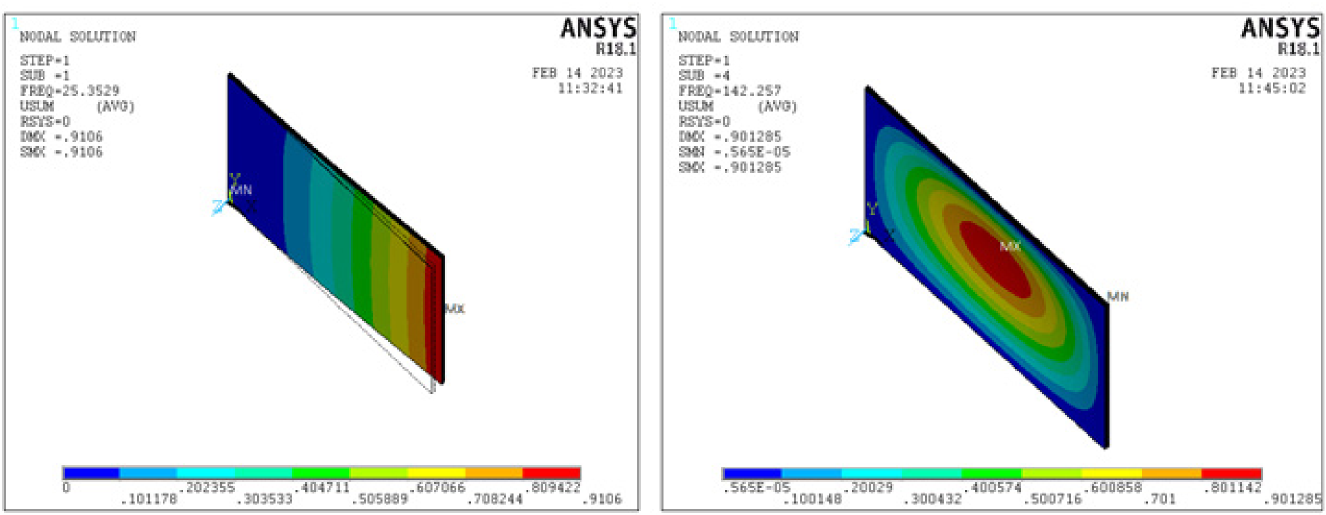
Tables 6 and Table 7 indicate that when k increases, the natural frequency of the FGM plate increases. Figure 6 show the comparison of first mode shape of FGM Plate with piezoelectric layer for three different boundary condition. The plot shows that the natural frequency of a clamped plate is substantially higher than the natural frequency of a plate that is only simply-supported.

The first mode shape of FGM Plate with Piezoelectric layer for Cantilevered and Simply-supported boundary conditions with exponent are shown in Figure 7.

The second mode shape of FGM Plate with Piezoelectric layer for Cantilevered and Simply-supported

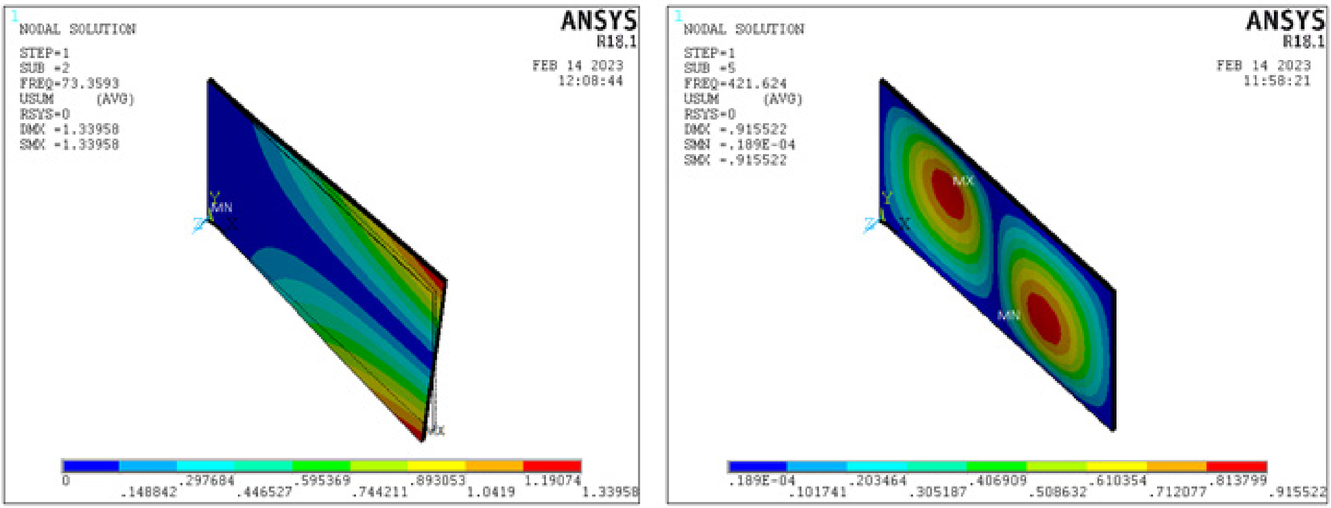
Table 7. Natural frequency (Hz) for the actuator layers that are bonded to the top and bottom surfaces of the simply supported FGM plates.

Mode no	k= 0	k = 0.2	k = 0.5	k = 1	k = 15	k = 100
1	142.20	168.45	185.23	198.50	247.10	259.25
2	359.00	421.62	462.46	495.45	615.45	645.35
3	359.00	421.62	462.46	495.45	615.45	645.35
4	563.95	664.86	731.01	778.54	967.35	1014.50
5	717.50	841.25	925.05	993.00	1231.00	1290.46



(a) (b)

Figure 7. The first Mode shape of FGM Plate with Piezoelectric layer for different edge conditions with exponent : (a) Cantileverd (b) Simply supported.



(a) (b)

Figure 8. The second Mode shape of FGM Plate with Piezoelectric layer for different boundary conditions with exponent : (a) Cantileverd (b) Simply supported

boundary conditions with exponent are shown in Figure 8.

6.3 FGM Plate with Porosity

In this we considered a simply supported functionally graded porous materials plate of lentgth (a) and width (b) each of 1 m and thickness (h) = 0.05 m, k = 0.1 is considered. Two piezoelectric layer of each thickness (h_p) equals to 0.1h. Figure 9 shows the first six mode shape of functionally graded porous materials plate. The electric conditions taken into account in this analysis are the closed-circuit condition and the open-circuit condition. In the closed-circuit condition, the electric potential is zero whereas in open-circuit condition, electric displacement is zero.

The first frequencies of the all Side Simply-Supported Square (SSSS) functionally graded porous materials plate (SUS304/Si_3N_4) plates are then shown in Table 8 for various power-law index k values. It is observed from Table 8, increase in value of k leads to decrease

Table 8. The first natural frequency of SSSS square functionally graded porous materials plate (φ =0.3, h=0.1 m)

Piezo layer thickness (h_p)	Electric condition	Natural frequency (Hz)				
		k =0	k =0.5	k =1	k =4	k =10
h_p=0.05h	Closed	950.72	711.02	639.56	550.13	519.07
	Open	961.02	719.63	647.92	557.62	526.07
h_p=0.10h	Closed	874.78	688.51	628.61	550.49	522.79
	Open	891.49	704.54	644.25	564.64	536.09
h_p=0.15h	Closed	829.71	678.03	626.38	556.71	531.68
	Open	852.75	700.45	648.31	576.70	550.53

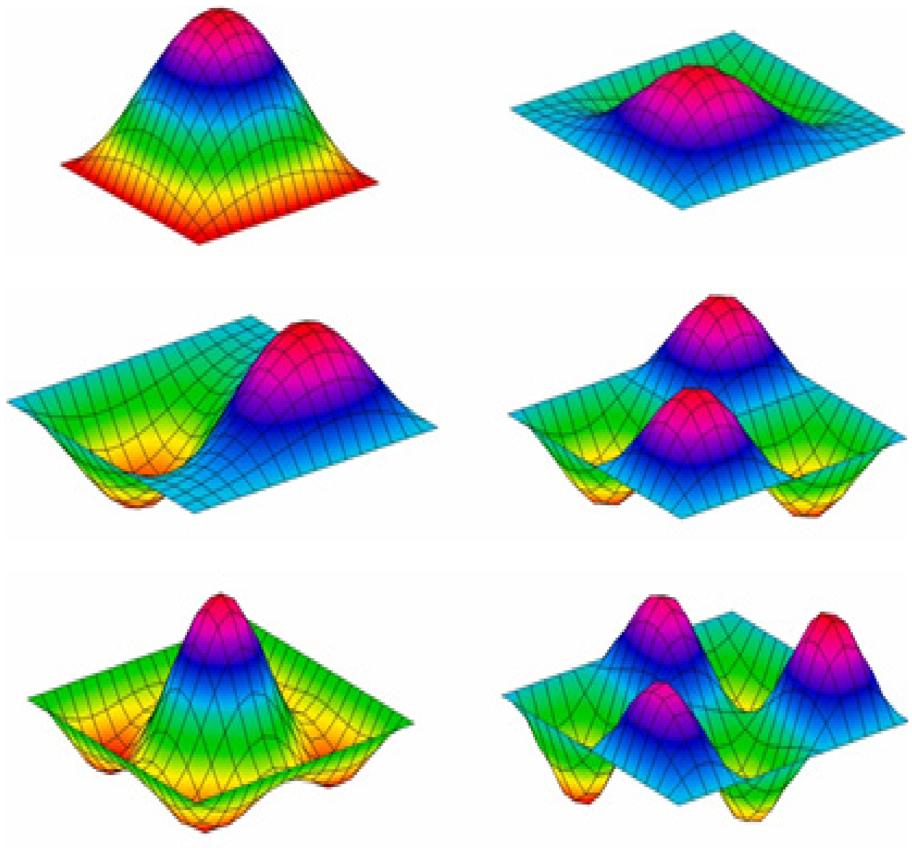


Figure 9. The six mode shapes of the SSSS square functionally graded porous plate with $\varphi = 0.1$ and $k = 0.1$.

in value of natural frequency. This is due to fact that as increase in k leads to decrease of plate stiffness very fast compare to mass. The natural frequency of plates is furthermore substantially influenced by the thickness of the piezoelectric layer. In the case of simply-supported plates on all sides, the natural frequency decreases as the thickness of the piezoelectric layers increases.

7.0 Conclusions

The free vibration analysis of FGM Plates integrated with piezoelectric layers is studied with and without porosity using the finite element method. For FGM Plate, the material characteristics of the FG porous plate are supposed to vary smoothly through the thickness direction as stated by power law embedded with porosity factor (φ). The following key conclusions can be inferred from the results:

- With increase in power law exponent, the FGM plate has higher natural frequencies.
- All edge clamped plate's natural frequency is significantly higher than that of a plate that is only all edge simply-supported . and it decreased as per order of the natural frequency follows fully clamped > simply supported > cantilevered.
- For functionally graded porous plate, the porosity φ and the power-law index k both result to reduce plate stiffness.

The frequecy results and mode shapes of this study may allow for the selection of a power law index-based grading system depending on the specific application.

8.0 References

1. Wang Z, Chen SH, Han W. The static shape control for intelligent structures. Finite Elements in Analysis and Design. 1997 Aug 15; 26(4):303-14.

2. Thai HT, Choi DH. A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates. Composite Structures. 2013 Jul 1; 101:332-40.
3. Tu TM, Quoc TH, Van Long N. Vibration analysis of functionally graded plates using the eight-unknown higher order shear deformation theory in thermal environments. Aerospace Science and Technology. 2019 Jan 1; 84:698-711.
4. Abrate S. Free vibration, buckling, and static deflections of functionally graded plates. Composites Science and Technology. 2006 Nov 1; 66(14):2383-94.
5. Kumar S, Jana P. Accurate solution for free vibration behaviour of stepped FGM plates implementing the dynamic stiffness method. In: Structures 2022 Nov 1 (Vol. 45, pp. 1971-1989). Elsevier.
6. Kumar S, Ranjan V, Jana P. Free vibration analysis of thin functionally graded rectangular plates using the dynamic stiffness method. Composite Structures. 2018 Aug 1; 197:39-53.
7. Kumar S, Jana P. Application of dynamic stiffness method for accurate free vibration analysis of sigmoid and exponential functionally graded rectangular plates. International Journal of Mechanical Sciences. 2019 Nov 1; 163:105105.
8. Kumar R, Jana P. Free vibration analysis of uniform thickness and stepped P-FGM plates: A FSDT-based dynamic stiffness approach. Mechanics Based Design of Structures and Machines. 2022 Aug 26; 1-30.
9. Kumar S, Gupta A, Shanmukhasundaram VR, Srivastava JP, Choudhary P. Determining the accurate free vibration response of thin functionally graded plates using the dynamic stiffness method. In: Functionally Graded Structures: Modelling and computation of static and dynamical problems 2023 Aug 1 (pp. 10-1). Bristol, UK: IOP Publishing
10. Hu Z, Shi Y, Xiong S, Zheng X, Li R. New analytic free vibration solutions of non-Lévy-type porous FGM rectangular plates within the symplectic framework. Thin-Walled Structures. 2023 Apr 1; 185:110609.
11. Jafarinezhad M, Sburlati R, Cianci R. Static and free vibration analysis of functionally graded annular plates using stress-driven nonlocal theory. European Journal of Mechanics-A/Solids. 2023 May 1; 99:104955.
12. Narayanan NI, Banerjee S. Free and Forced Vibration Analysis of FGM Plates with and Without Cutouts Using Heterosis Finite Element Method. Journal of Vibration Engineering & Technologies. 2023 Apr 30; 1-7.
13. Jana K, Pal S, Haldar S. Modal analysis of power law functionally graded material plates with rectangular cutouts. Mechanics Based Design of Structures and Machines. 2023 Feb 17; 1-29.
14. Sah SK, Ghosh A. Free vibration and buckling analysis of FGM plates using inverse trigonometric shear deformation theory. Aircraft Engineering and Aerospace Technology. 2021 Apr 5; 93(2):298-310.
15. Parida S, Mohanty SC. Vibration analysis of FG rotating plate using nonlinear-FEM. Multidiscipline Modeling in Materials and Structures. 2018 Aug 7; 15(1):26-49.
16. Tran MT, Thai S. Transient analysis of variable thickness multi-directional functionally graded plates using isogeometric analysis. Multidiscipline Modeling in Materials and Structures. 2023 Jun 5; 19(4):652-79.
17. Huang XL, Shen HS. Vibration and dynamic response of functionally graded plates with piezoelectric actuators in thermal environments. Journal of Sound and Vibration. 2006 Jan 3; 289(1-2):25-53.
18. Liu GR, Dai KY, Han X, Ohyoshi T. Dispersion of waves and characteristic wave surfaces in functionally graded piezoelectric plates. Journal of Sound and Vibration. 2003 Nov 13; 268(1):131-47.
19. Talha M, Singh B. Static response and free vibration analysis of FGM plates using higher order shear deformation theory. Applied mathematical modelling. 2010 Dec 1; 34(12):3991-4011.
20. Rezaei AS, Saidi AR, Abrishamdari M, Mohammadi MP. Natural frequencies of functionally graded plates with porosities via a simple four variable plate theory: an analytical approach. Thin-Walled Structures. 2017 Nov 1; 120:366-77.
21. Wang YQ, Wan YH, Zhang YF. Vibrations of longitudinally traveling functionally graded material plates with porosities. European Journal of Mechanics-A/Solids. 2017 Nov 1; 66:55-68.
22. Wang YQ, Zu JW. Vibration behaviors of functionally graded rectangular plates with porosities and moving in thermal environment. Aerospace Science and Technology. 2017 Oct 1; 69:550-62.
23. Shahsavari D, Shahsavari M, Li L, Karami B. A novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation. Aerospace Science and Technology. 2018 Jan 1; 72:134-49.
24. Bansal G, Gupta A, Katiyar V. Vibration of porous functionally graded plates with geometric discontinuities and partial supports. Proceedings of the Institution of

Mechanical Engineers, Part C: Journal of Mechanical Engineering Science. 2020 Nov; 234(21):4149-70.

25. Kim J, Żur KK, Reddy JN. Bending, free vibration, and buckling of modified couples stress-based functionally graded porous micro-plates. Composite Structures. 2019 Feb 1; 209:879-88.
26. Huang XL, Shen HS. Nonlinear vibration and dynamic response of functionally graded plates in thermal environments. International Journal of Solids and Structures. 2004 May 1; 41(9-10):2403-27.
27. Farsangi MA, Saidi AR. Levy type solution for free vibration analysis of functionally graded rectangular plates with piezoelectric layers. Smart materials and structures. 2012 Aug 31; 21(9):094017.