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Ferroconvection in a horizontal Darcy-Brinkman porous medium with radiative transfer

The classical linear stability analysis is used to examine the effect of thermal radiation on the onset of Darcy-Brinkman ferroconvection. The boundaries of the fluid layer are treated as black bodies and the optical properties of the transparent ferromagnetic fluid are independent of the wave length of radiation. The fluid and solid matrix are assumed to be in local thermal equilibrium. Considering realistic boundary conditions, the principle of exchange of stabilities is shown to be valid and the critical values pertaining to the stationary instability are obtained by means of the higher order Galerkin method. It is observed that the basic temperature profile turns out to be exponential and symmetric as the radiative parameters increase and that the effect of thermal radiation is to delay the onset of Darcy-Brinkman ferroconvection. The destabilizing influence of magnetic forces is affected by the radiative parameters. The effect of magnetic, radiative and porous parameters on the convection cell size is also discussed.

Keywords: *Ferrofluid, thermal radiation, porous media, magnetic field*

1.0 Introduction

Combined heat transfer processes such as convection-radiation play a significant role in several chemical processes involving combustion, drying, fluidization, MHD flows, and so forth. The complexity involved in the solution of the integro-differential equations resulting from the coupled convection and radiation problem warrants the use of several simplifying assumptions. In general, the radiative process either occurs at the boundaries or as a term in the energy equation. In the latter case, the radiative term is usually approximated as a flux in such a way that the term corresponding to radiation in the heat transfer equation appears as a gradient term similar to Fourier's conduction term. This method has found considerable favour among many researchers. Alternatively, radiation effects can be incorporated at the boundaries through appropriate assumptions. Free surface flows present a challenging problem to engineers as the combined convection-radiation at the boundaries has major applications in many industries.

Goody (Goody, 1956) estimated the radiative transfer effects in the conventional natural convection problem with free boundaries using a variational method. He solved the problem for optically thin and optically thick cases and showed that there could be very large variations near the boundaries. Goody's radiative transfer model has been extended and modified by subsequent investigators to take into account the effects of magnetic field, rotation and fluid non-grayness (Spiegel, 1960; Murgai and Khosla, 1962; Khosla and Murgai, 1963; Christophorides and Davis, 1970; Arpaci and Gozum, 1973; Yang, 1990; Bdeoui and Soufiani, 1997).

Larson (Larson, 2001) studied linear and nonlinear stability properties of Goody's model analytically. When thermal diffusivity is zero, the energy method is used to rule out subcritical instabilities. When thermal diffusivity is nonzero, the energy method is used to find a critical threshold below which all infinitesimal and finite amplitude perturbations are stable.

Shobha Devi et al. (Shobha Devi et al., 2002) studied the problem of Rayleigh-Bénard convection in an anisotropic porous medium in the presence of radiation. A linear stability analysis is performed and the Milne-Eddington approximation is employed for obtaining the initial static state. The Galerkin method is used to obtain the critical Rayleigh numbers. It is shown that radiation is to stabilize the system for both transparent and opaque media. It is found that opaque media releases heat for convection more slowly than transparent media and that the cell size gets affected by radiation only in the case of transparent media.

Maruthamanikandan (Maruthamanikandan, 2003) analyzed the effect of radiative transfer on the onset of thermal convection in a ferromagnetic fluid layer bounded by two parallel plates and heated from below. The Milne-Eddington approximation is employed to convert radiative heat flux into thermal heat flux. It is found that radiation inhibits the onset

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of convection in both transparent and opaque media. Furthermore, the opaque medium is shown to release heat for convection more slowly than the transparent medium. It is also shown that radiation affects the cell size at the onset of convection only in the case of transparent medium.

Anwar et al. (Anwar et al., 2008) studied the effects of thermal radiation and porous drag forces on the natural convection heat and mass transfer of a viscous, incompressible, gray, absorbing, emitting fluid flowing past an impulsively started moving vertical plate adjacent to a non-Darcian porous regime. The Rosseland diffusion approximation is employed to analyze the radiative heat flux and is appropriate for non-scattering media. Increasing Darcy number is seen to accelerate the flow; the converse is apparent for an increase in Forchheimer number. Thermal radiation is seen to reduce both velocity and temperature in the boundary layer.

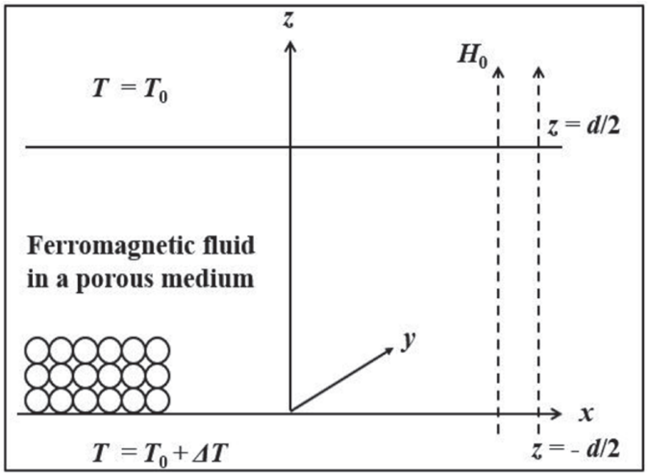
Shateyi et al. (Shateyi et al., 2010) sought to investigate the influence of a magnetic field on heat and mass transfer by mixed convection from vertical surfaces in the presence of Hall, radiation, Soret thermal diffusion, and Dufour diffusion-thermo effects. Similarity solutions were obtained using suitable transformations. The numerical results for some special cases were compared to the exact solution and were found to be in good agreement.

Jafarunnisa et al. (Jafarunnisa et al., 2012) analyzed the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel in the presence of heat generating sources. The nonlinear coupled governing equations are solved by a regular perturbation technique. The effect of chemical reaction and radiation absorption on all flow characteristics are discussed with the help of graphs.

Most investigations of ferroconvection consider only thermally conducting ferromagnetic fluids (Maruthamanikandan et al., 2018; Soya Mathew and Maruthamanikandan, 2018; Nisha Mary Thomas and Maruthamanikandan, 2013; 2018; 2020) albeit the research concerning light absorption by magnetic fluids based on petroleum showed that magnetic fluids can be utilized as absorbent media for solar energy. Under these circumstances, in this paper, we extend Goody's model to take into account the magnetic force and porous medium. In fact, we study qualitatively the effect of thermal radiative transfer on the onset of ferroconvection in a porous medium in the presence of a uniform vertical magnetic field. We also restrict our attention to the case in which the absorption coefficient of the fluid is the same at all wave lengths and is independent of the physical state (the so-called gray medium approximation). The equation of radiative transfer is developed in optically thin approximation and the effect of scattering is ignored. The results are illustrated graphically.

2.0 Mathematical formulation

Consider a horizontal constant porosity layer of a ferromagnetic fluid confined between two parallel infinite boundaries heated from below. The boundaries are assumed to be perfect conductors of heat. A Cartesian coordinate system is used with the z-axis vertically upward. The lower surface at $z=-d/2$ and upper surface at $z=d/2$ are maintained at constant temperatures T_1 and T_o. The fluid between the boundaries absorbs and emits thermal radiation. We treat the two boundaries as black bodies. The absorption coefficient of the fluid is assumed to be the same at all wavelengths and to be independent of the physical state. Moreover, it is assumed that local thermal equilibrium exists between the solid matrix and the saturated fluid.



Schematic of the problem

The system of equations describing the problem at hand is the following

$$\nabla \cdot \vec{q} = 0 \quad \text{... (1)}$$

$$\rho_o \left[\frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} - \frac{\mu_f}{k} \vec{q} + \bar{\mu}_f \nabla^2 \vec{q} + \nabla \cdot (\vec{H}\vec{B}) \quad \text{... (2)}$$

$$\varepsilon C_f \left[\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right] + (1 - \varepsilon)(\rho_o C)_s \frac{\partial T}{\partial t} + \mu_o T \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,H} \cdot \left[\frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} \right] = k_1 \nabla^2 T + C_f \frac{G}{s_r} \quad \text{... (3)}$$

$$\rho = \rho_o [1 - \alpha (T - T_a)] \quad \text{... (4)}$$

where $\vec{q} = (u, v, w)$ is the fluid velocity, ρ_0 is the reference density, ε is the porosity of the porous medium, t is the time, p is the pressure, ρ is the fluid density, $\vec{g}$ is the acceleration due to gravity, μ_f is the dynamic viscosity, $\bar{\mu}_f$ is the effective viscosity, k is the permeability of the porous medium, $\vec{H}$ is the magnetic field, $\vec{B}$ is the magnetic induction, T is the temperature, α is the thermal expansion coefficient, T_a the arithmetic mean of boundary temperatures, σ_o is the magnetic permeability, $\vec{M}$ is the magnetization, k_1 is the thermal conductivity, G is the rate of radiative heating per unit volume, s_r is the heat content of the fluid per unit volume, ∇ is the vector differential operator, (x, y, z) are the spatial coordinates, $C_f = \rho_o C_{V,H} - \mu_o \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}$ and $C_{V,H}$ is the specific heat at constant volume and constant magnetic field.

The relevant Maxwell equations are

$$\nabla \cdot \vec{B} = 0 \qquad ... (5)$$

$$\nabla \times \vec{H} = \vec{0} \qquad ... (6)$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) \qquad ... (7)$$

Since the magnetization is aligned with the magnetic field and is a function of temperature and magnetic field, we may write

$$\vec{M} = \frac{\vec{H}}{H} M\left(H, T\right) \qquad ... (8)$$

The magnetic equation of state is linearized about the magnetic field and the reference temperature to become

$$M = M_0 + \chi\left(H - H_0\right) - K\left(T - T_a\right) \qquad ... (9)$$

where χ is the magnetic susceptibility and K is the pyromagnetic coefficient.

The equation of radiative heat transfer is

$$\frac{dI(\vec{s})}{ds} = K_a\left[P_B - I(\vec{s})\right] \qquad ... (10)$$

where $I(\vec{s})$ is the intensity of radiation along the direction of the vector $\vec{s}$, ds is an infinitesimal displacement in the $\vec{s}$ direction, K_a is the absorption coefficient of the fluid and P_B is the Planck black-body intensity. The radiative heating rate is given by

$$G = -\int \frac{dI(\vec{s})}{ds} d\omega_s \qquad (11)$$

where the integral is taken over the solid angle and ωs is the element of solid angle.

The basic state is quiescent and is described by

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b = (0,0,0),\ \rho = \rho_b(z), T = T_b(z), \\ &p = p_b(z),\ \vec{H} = \vec{H}_b = \left(0,0,H_b(z)\right), \\ &M = \vec{M}_b = \left(0,0,M_b(z)\right),\ G = G(z) \end{aligned}\right\} \qquad ... (12)$$

where the subscript b denotes the basic state.

The quiescent basic state has a solution in the form

$$\rho_b = \rho_0\left[1 + \alpha\beta z\right] \qquad ... (13)$$

$$\vec{H}_b = \left[H_0 - \frac{K\beta z}{1+\chi}\right]\hat{k} \qquad ... (14)$$

$$\vec{M}_b = \left[M_0 + \frac{K\beta z}{1+\chi}\right]\hat{k} \qquad ... (15)$$

$$\vec{B}_b = \mu_0\left(\vec{H}_0 + \vec{M}_0\right)\hat{k} \qquad ... (16)$$

where H_o is the uniform magnetic field, M_o is the reference magnetization and $\beta = \frac{T_1 - T_0}{d}$. In the quiescent basic state, the equation (10) of radiative transfer takes the form

$$\mu_3 \frac{dI}{dz} = K_a\left[P_B - I\right] \qquad ... (17)$$

where μ_3 is the directional cosine of $\vec{s}$ in the z-direction. Equation (17) explains the fact that the intensity of radiation is increased by emission and decreased by absorption.

In the basic state the energy equation (3) becomes

$$\frac{G_b}{s_r} + \kappa \frac{d^2 T_b}{dz^2} = 0 \qquad ... (18)$$

where $\kappa = \frac{k_1}{C_f}$. Equation (18) is suggestive of the fact that the heat transfer in the basic state is essentially by conduction and radiation. If F_Z is the z-component of the radiative heat flux, then we may write

$$G_b = -\frac{dF_z}{dz} \qquad ... (19)$$

and we may write Eq. (18) in the integrated form

$$F_z + \kappa s_r \beta = C_1 \qquad ... (20)$$

where C_1 is the constant of integration.

Iterative solutions of one-dimensional radiative equilibrium problems all show that remarkably accurate results can be obtained by assuming a simple form for the angular distribution of radiative intensity. Assuming the Milne-Eddington approximation (Goody, 1956), and using the radiative heat transfer equation (17), the differential equation associated with the heat flux can be obtained in the form

$$\frac{d^2 F_z}{dz^{*2}} - \lambda^2 F_z = -\lambda^2 \frac{\psi}{1+\psi} C_1 \qquad ... (21)$$

where $z^* = \frac{z}{d}, \lambda^2 = 3K_a^2 d^2\left(1+\psi\right), \psi = \frac{4\pi Q}{3\kappa K_a s_r}$, $Q = \frac{4\sigma_s}{\pi} T_0^3$, and σ_s is the Stefan-Boltzmann constant. Solving Eq. (21) using the following dimensionless radiative boundary conditions

$$\left.\begin{aligned} &\frac{dF_z}{dz^*} = -2K_a F_z d \quad \text{at} \quad z^* = +1/2 \\ &\frac{dF_z}{dz^*} = +2K_a F_z d \quad \text{at} \quad z^* = -1/2 \end{aligned}\right\} \qquad ... (22)$$

we obtain the following solution

$$f(z^*) = \frac{\beta}{\bar{\beta}} = L_1 \cosh(\lambda z^*) + L_2 \qquad \text{... (23)}$$

where $L_1 = \psi \left[\frac{2\psi}{\lambda} + \frac{1}{2}\sqrt{3+3\psi}\, \sinh\left(\lambda/2\right) + \cosh\left(\lambda/2\right) \right]^{-1}$

and $L_2 = \frac{L_1}{\psi}\left[\frac{1}{2}\sqrt{3+3\psi}\, \sinh\left(\lambda/2\right) + \cosh\left(\lambda/2\right) \right]$ and $\bar{\beta}$ is the mean value of β throughout the medium. The radiative boundary conditions (22) are obtained using the fact that the molecular conduction ensures continuity of temperature at the two surfaces. It is advantageous mentioning that $f(z^*)$ tends to unite if either λ or ψ tends to zero independently. Moreover, if λ and ψ are both greater than unity, the variation of the basic state temperature is exponential. In other words, the basic state temperature is no longer linear if the radiation effect is accounted for. In what follows we study the stability of the quiescent state within the framework of the linear theory

3.0 Stability analysis

Let the components of the perturbed physical quantities be

$$\left.\begin{array}{l} \vec{q} = \vec{q}_b + \vec{q}' = (u', v', w'),\, p = p_b + p', \\ \rho = \rho_b + \rho',\, T = T_b + T',\, \vec{M} = \vec{M}_b + \vec{M}', \\ \phi = \phi_b + \phi',\, \vec{H} = \vec{H}_b + \vec{H}',\, G = G_b + G' \end{array}\right\} \qquad \text{... (24)}$$

where the primes indicate infinitesimally small perturbations $\vec{H} = \nabla\phi'$ and with ϕ' being the perturbed magnetic potential. Substituting (24) into the governing equations, neglecting the nonlinear terms, incorporating the quiescent state solutions and eliminating the pressure term gives the following equations

$$\rho_o \frac{\partial}{\partial t}\left(\nabla^2 w'\right) = \alpha \rho_o g \nabla_1^2 T' - \frac{\mu_f}{k}\nabla^2 w' + \frac{\mu_o K^2 \beta}{1+\chi}\nabla_1^2 T' + \bar{\mu}_f \nabla^4 w' - \mu_o K \beta \frac{\partial}{\partial z}\left(\nabla_1^2 \phi'\right) \qquad \text{... (25)}$$

$$(\rho_o C)\frac{\partial T'}{\partial t} + \left[\frac{\mu_o K^2 T_a \beta}{1+\chi} - C_f \beta\right] w' - \mu_o K T_a \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = k_1 \nabla^2 T' + C_f \frac{G'}{s_r} \qquad \text{... (26)}$$

$$(\rho_o C)\frac{\partial T'}{\partial t} + \left[\frac{\mu_o K^2 T_a \beta}{1+\chi} - C_f \beta\right] w' - \mu_o K T_a \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = k_1 \nabla^2 T' + C_f \frac{G'}{s_r} \qquad (26)$$

$$\left(1 + \frac{M_o}{H_o}\right)\nabla_1^2 \phi' + (1+\chi)\frac{\partial^2 \phi'}{\partial z^2} - K\frac{\partial T'}{\partial z} = 0 \qquad \text{... (27)}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Since Eq. (26) is an integro-differential equation, we adopt the approximation which is valid when the fluid medium is optically thin (known as transparent approximation). For the transparent approximation, the relation becomes (Goody, 1956)

$$\nabla_1^2 G' = -4\pi Q K_a \nabla_1^2 T' \qquad \text{... (28)}$$

Equation (26) after making use of (28), becomes

$$(\rho_o C)\frac{\partial\left(\nabla_1^2 T'\right)}{\partial t} + \left[\frac{\mu_o K^2 T_a \beta}{1+\chi} - C_f \beta\right]\nabla_1^2 w' - \mu_o K T_a \frac{\partial}{\partial t}\nabla_1^2\left(\frac{\partial \phi'}{\partial z}\right) = k_1 \nabla^2\left(\nabla_1^2 T'\right) - \frac{4\pi Q K_a}{s_r} C_f \nabla_1^2 T' \qquad \text{... (29)}$$

The normal mode solution for the dependent variables is given by

$$\begin{bmatrix} w' \\ T' \\ \phi' \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{bmatrix} e^{i(lx + my) + \sigma t} \qquad \text{... (30)}$$

where l and m are the dimensionless wavenumbers in the x and y directions respectively and σ is the growth rate. Substitution of (30) into equations (25), (29) and (27) leads to

$$\rho_o \sigma\left(D^2 - k_h^2\right)W = -\alpha \rho_o g k_h^2 \Theta - \frac{\mu_f}{k}\left(D^2 - k_h^2\right)W + \bar{\mu}_f\left(D^2 - k_h^2\right)^2 W - \frac{\mu_o K^2 \beta}{1+\chi}k_h^2 \Theta + \mu_o K \beta k_h^2 D\Phi \qquad \text{... (31)}$$

$$(\rho_o C)\sigma\Theta + \left[\frac{\mu_o K^2 T_a \beta}{1+\chi} - C_f \beta\right]W - \mu_o K T_a \sigma D\Phi = k_1\left(D^2 - k_h^2\right)\Theta - \frac{4\pi Q K_a}{s_r}C_f \Theta \qquad \text{... (32)}$$

$$(1+\chi)D^2\Phi - \left(1 + \frac{M_o}{H_o}\right)k_h^2 \Phi - K D\Theta = 0 \qquad \text{... (33)}$$

where $D = \frac{d}{dz}$ and $k_h^2 = l^2 + m^2$ is the overall horizontal wavenumber. Non-dimensionalizing equations (31) - (33) using the transformations

$$W^* = \frac{Wd}{\kappa},\ \Phi^* = \frac{\Phi(1+\chi)}{K\bar{\beta}d^2},\ \sigma^* = \frac{\sigma d^2}{\kappa},\quad \Theta^* = \frac{\Theta}{\bar{\beta}d},\ z^* = \frac{z}{d},\quad a = k_h d \qquad \text{... (34)}$$

we obtain

$$\frac{\sigma}{Pr}\left(D^2-a^2\right)W = -(R+N)a^2\Theta - Da^{-1}\left(D^2-a^2\right)W + \Lambda\left(D^2-a^2\right)^2 W + Na^2 D\Phi \qquad \text{... (35)}$$

$$\lambda\sigma\Theta - M_2\sigma D\Phi = (1-M_2)f(z)W + \left[D^2 - a^2 - \lambda^2\frac{\psi}{1+\psi}\right]\Theta \qquad \text{... (36)}$$

$$\left(D^2 - M_3 a^2\right)\Phi - D\Theta = 0 \qquad \text{... (37)}$$

where $Pr = \frac{\mu_f}{\rho_0\kappa}$ is the Prandtl number, $Da^{-1} = \frac{d^2}{k}$ is the inverse Darcy number, $\Lambda = \frac{\bar{\mu}_f}{\mu_f}$ is the Brinkman number, $R = \frac{\alpha\rho_0 g\bar{\beta}d^4}{\mu_f\kappa}$ is the thermal Rayleigh number, $N = \frac{\mu_0 K^2\beta\bar{\beta}d^4}{\mu_f(1+\chi)\kappa}$ is the magnetic Rayleigh number and $M_3 = \frac{\left(1+\frac{M_0}{H_0}\right)}{(1+\chi)}$ is the non-buoyancy magnetization parameter.

The boundary conditions are (Finlayson, 1970)

$$\left.\begin{aligned} &W = DW = \Theta = 0 \quad \text{at} \quad z = \pm 1/2 \\ &D\Phi + \frac{a\Phi}{1+\chi} = 0 \quad \text{at} \quad z = 1/2 \\ &D\Phi - \frac{a\Phi}{1+\chi} = 0 \quad \text{at} \quad z = -1/2 . \end{aligned}\right\} \qquad \text{... (38)}$$

3.1 Stationary Instability

Stationary instability is characterised by $\sigma = 0$ and the associated equations, from Eqs. (35) - (37), are therefore given by

$$\Lambda\left(D^2-a^2\right)^2 W - Da^{-1}\left(D^2-a^2\right)W - (R+N)a^2\Theta + Na^2 D\Phi = 0 \qquad \text{... (39)}$$

$$f(z)W + \left(D^2 - a^2 - \lambda^2\frac{\psi}{1+\psi}\right)\Theta = 0 \qquad \text{... (40)}$$

$$\left(D^2 - M_3 a^2\right)\Phi - D\Theta = 0 \qquad \text{... (41)}$$

3.2 Oscillatory instability

We now examine the validity of the principle of exchange of stabilities (PES) for the problem at hand by means of the Galerkin method. Multiplying equations (35)-(37) by W, Θ and Φ respectively, integrating the resulting equations with respect z between the limits $z = -1/2$ and $z = 1/2$, taking $W(z) = A_1 W_1(z)$, $\Theta(z) = B_1\Theta_1(z)$ and $\Phi(z) = C_1\Phi_1(z)$ (in which W_1, Θ_1 and Φ_1 are trial functions) leads to the following system of equations:

$$\left[\left(\frac{\sigma}{Pr} + Da^{-1}\right)P_1 - \Lambda P_2\right]A_1 + (R+N)P_3 B_1 - NP_4 C_1 = 0 \qquad \text{... (42)}$$

$$P_5 A_1 + \left[P_7 - \frac{\lambda^2\psi}{1+\psi}P_6 - \sigma P_6\right]B_1 = 0 \qquad \text{... (43)}$$

$$P_8 B_1 - \left[P_9 - M_3 P_{10}\right]C = 0 \qquad \text{... (44)}$$

where

$$\begin{aligned} &P_1 = \langle W_1 D^2 W_1\rangle - a^2\langle W_1^2\rangle, \\ &P_2 = \langle W_1 D^4 W_1\rangle + a^4\langle W_1^2\rangle - 2a^2\langle W_1 D^2 W_1\rangle,\ P_3 = a^2\langle W_1\Theta_1\rangle, \\ &P_4 = a^2\langle W_1 D\Phi_1\rangle,\ P_5 = \langle\Theta_1 f(z)W_1\rangle, \\ &P_6 = \langle\Theta_1^2\rangle,\ P_7 = \langle\Theta_1 D^2\Theta_1\rangle - a^2\langle\Theta_1^2\rangle, \\ &P_8 = \langle\Phi_1 D\Theta_1\rangle,\ P_9 = \langle\Phi_1 D^2\Phi_1\rangle, \\ &P_{10} = a^2\langle\Phi_1^2\rangle \end{aligned}$$

and the inner product is defined as $\langle fg\rangle = \int_{1/2}^{-1/2} fg\,dz$. Assuming $\sigma = i\,\omega$ with ω being the frequency of oscillations, the criterion for the existence of the unique solution of the system of equations (42)-(44) leads to the expression

$$R = R_1 + iR_2$$

where

$$R_1 = \frac{\left[\begin{aligned} &M_3 P_{10}\left\{\begin{aligned} &-N\,Pr\,P_3P_5(1+\psi) + \\ &Pr\left(Da^{-1}P_1 - \Lambda P_2\right) \\ &\left(P_7 + \psi P_7 - \lambda^2\psi P_6\right) \end{aligned}\right\} \\ &+ P_1P_6(1+\psi)\omega^2 - P_1P_6P_9(1+\psi)\omega^2 \\ &-Pr\left\{\begin{aligned} &NP_5(P_4P_8 - P_3P_9)(1+\psi) + \\ &P_9\left(Da^{-1}P_1 - \Lambda P_2\right) \\ &\left(P_7 + \psi P_7 - \lambda^2\psi P_6\right) \end{aligned}\right\} \end{aligned}\right]}{Pr\,P_3P_5(1+\psi)(M_3P_{10} - P_9)}$$

and

$$R_2 = \frac{\begin{bmatrix} \Lambda Pr P_2 P_6 (1+\psi) + \\ P_1 \left(P_7 + \psi P_7 - \lambda^2 \psi P_6 \right) \\ - Da^{-1} Pr P_1 P_6 (1+\psi) \end{bmatrix} \omega}{Pr P_3 P_5 (1+\psi)}$$

As the Rayleigh number R is real, we must have R_2=0. Clearly ω=0. This means that the PES is valid for the present problem and the possibility of existence of oscillatory instability is ruled out. Hence the stationary instability is the preferred mode.

4.0 Method of solution

The system comprising Eqs. (39)-(41) and the homogeneous boundary conditions (38) is an eigenvalue problem with being the eigenvalue. An approximate solution of this eigenvalue problem can be obtained by the well-known Galerkin method (Finlayson, 1972). To this end, we let

$$W = \sum A_i W_i, \quad \Theta = \sum B_i \Theta_i, \quad \Phi = \sum C_i \Phi_i$$

where A_i, B_i and C_i are constants and the basis functions Wi, Θ_i and Φ_i are represented by a power series satisfying the respective boundary conditions. Appealing to the Galerkin method which demands the residuals be orthogonal to the following system of homogenous algebraic equations

$$\left. \begin{aligned} D_{ji} A_i + E_{ji} B_i + F_{ji} C_i &= 0 \\ G_{ji} A_i + H_{ji} B_i &= 0 \\ K_{ji} B_i + L_{ji} C_i &= 0 \end{aligned} \right\} \qquad \ldots (45)$$

where

$$D_{ji} = \Lambda \begin{bmatrix} < W_j D^4 W_i > + a^4 < W_j W_i > \\ - 2a^2 < W_j D^2 W_i > \end{bmatrix}$$

$$- Da^{-1} \left[< W_j D^2 W_i > - a^2 < W_j W_i > \right]$$

$$E_{ji} = -(R+N) a^2 < W_j \Theta_i >,$$

$$F_{ji} = N a^2 < W_j D\Phi_i >, G_{ji} = < \Theta_j f(z) W_i >$$

$$H_{ji} = < \Theta_j D^2 \Theta_i > - a^2 < \Theta_j \Theta_i >$$

$$- \lambda^2 \frac{\psi}{1+\psi} < \Theta_j \Theta_i >, K_{ji} = < \Phi_j D\Theta_i >$$

$$L_{ji} = < D\Phi_j D\Phi_i > + M_3 a^2 < \Phi_j \Phi_i >$$

$$+ \frac{a}{1+\chi} \begin{bmatrix} \Phi_j (1/2) \Phi_i (1/2) \\ + \Phi_j (-1/2) \Phi_i (-1/2) \end{bmatrix}.$$

The trial functions chosen are $W_i = \left(z^2 - \frac{1}{4} \right)^{i+1}$, $\Theta_i = \left(z^2 - \frac{1}{4} \right)^{i}$ and $\Phi_i = z^{2i-1}$. and. On applying the Galerkin method to the system (45) of equations, we would obtain the critical Rayleigh number and the corresponding critical wavenumber.

5.0 Results and discussions

The influence of thermal radiation on the onset of Darcy-Brinkman ferroconvection in an absorbing and emitting ferromagnetic fluid layer in the presence of a vertical uniform magnetic field is studied. The boundaries are assumed black bodies and the optical properties of the transparent ferromagnetic fluid are independent of the wave length of radiation. It is assumed that the fluid and solid matrix are in local thermal equilibrium. Realistic hydrodynamic boundary conditions and general boundary conditions on the magnetic potential are considered. The principle of exchange of stabilities is shown to be valid by means of the single term Galerkin method. The critical values pertaining to the stationary instability are obtained using the higher order Galerkin method. As regards the values of radiative parameters ψ and λ, it is to be noted that large radiative effects are more likely if a gas rather than a liquid is used as a fluid. In view of this, large values of ψ and λ have been overlooked in the problem at hand.

To get a better understanding of the results obtained, we examine the basic state temperature distribution which throws some light on the effect of radiative heat transfer on the stability of the system. Figs.1 and 2 are plots of z versus $f(z)$ for different values of ψ and λ respectively. We observe that the basic state temperature profile becomes exponential and nonlinear as ψ and λ increase and it is symmetric about

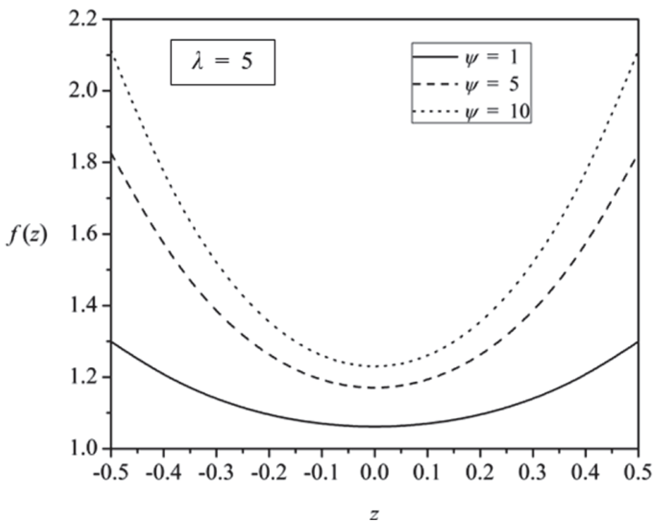


Fig.1: Basic temperature profiles for different values of the conduction-radiation parameter ψ.

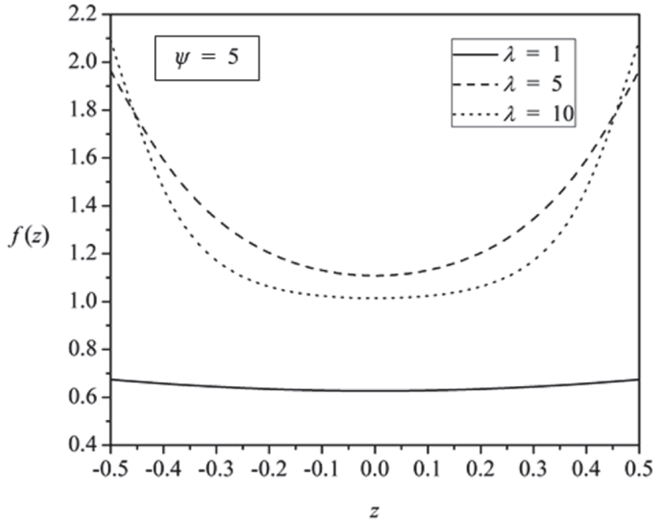


Fig.2: Basic temperature profiles for different values of the absorptivity parameter λ.

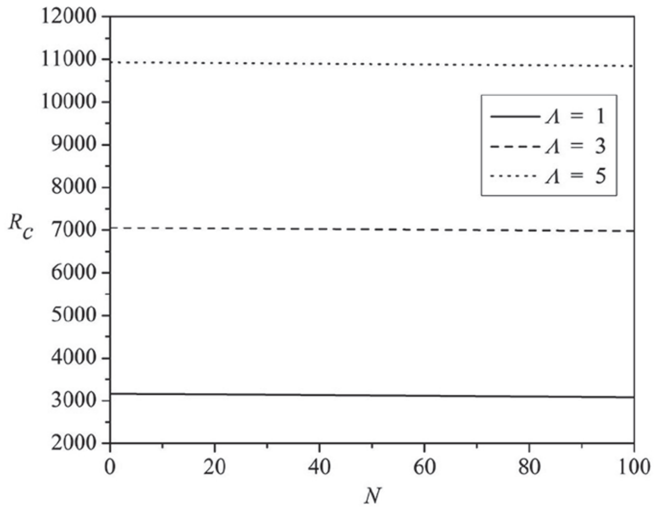


Fig.3: Plot of R_c as a function of N for different values of Λ and Da^{-1}=5, ψ=5, λ=5, M_3=3, χ=2.

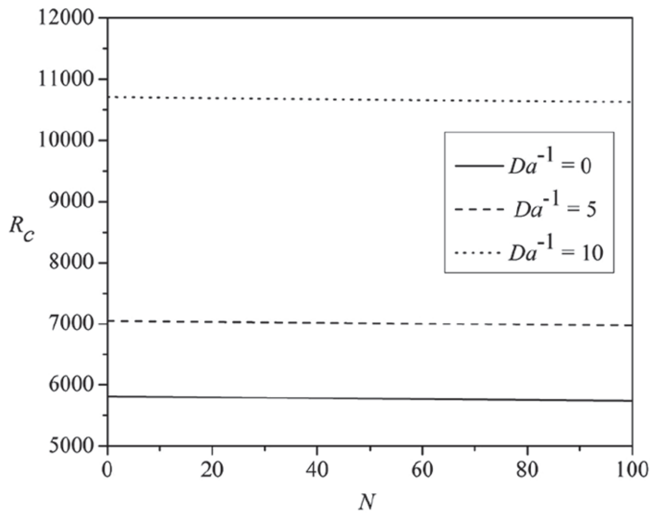


Fig.4: Plot of R_c as a function of N for different values of Da^{-1} and Λ=3, ψ=5, λ=5, M_3=3, χ=2.

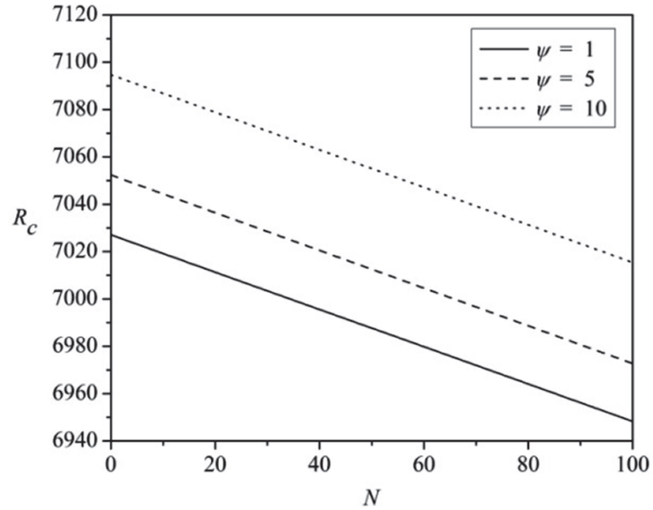


Fig.5: Plot of R_c as a function of N for different values of ψ and Da^{-1}=5, Λ=3, λ=5, M_3=3, χ=2.

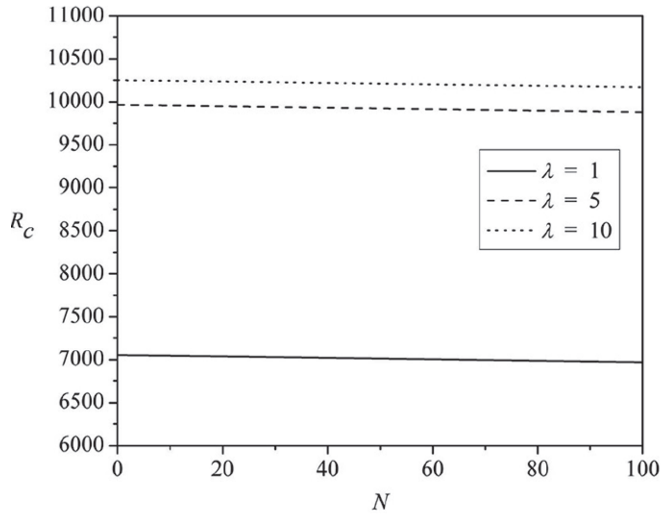


Fig.6: Plot of R_c as a function of N for different values of λ and Da^{-1}=5, Λ=3, ψ=5, M_3=3, χ=2.

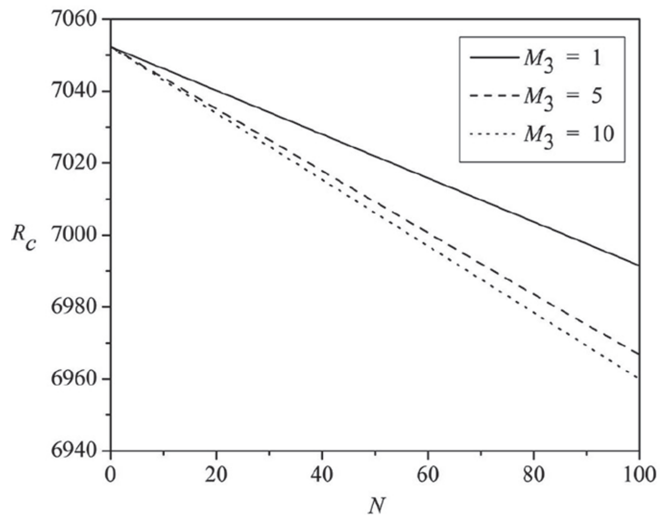


Fig.7: Plot of R_c as a function of N for different values of M_3 and Da^{-1}=5, Λ=3, ψ=5, λ=5, χ=2.

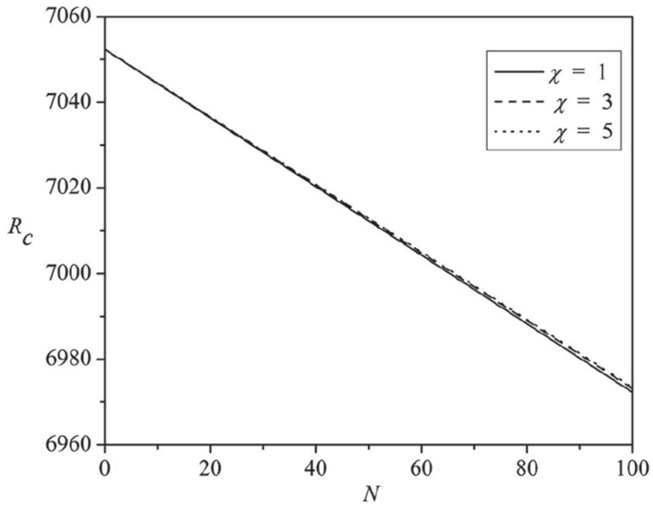


Fig.8: Plot of R_c as a function of N for different values of χ and Da^{-1}=5, Λ=3, ψ=5, λ=5, M_3=3

TABLE 1: VARIATION OF a_c WITH N FOR DIFFERENT VALUES OF Λ AND Da^{-1}=5, ψ=5, λ=5, M_3=3, χ=2.

N	a_c		
	Λ = 1	Λ = 3	Λ = 5
0	3.508	3.445	3.428
20	3.510	3.446	3.429
40	3.513	3.447	3.429
60	3.515	3.448	3.430
80	3.517	3.449	3.431
100	3.520	3.450	3.431

TABLE 2: VARIATION OF a_c WITH N FOR DIFFERENT VALUES OF Da^{-1} AND Λ=3, ψ=5, λ=5, M_3=3, χ=2.

N	a_c		
	Da^{-1}=0	Da^{-1}=5	Da^{-1}=10
0	3.398	3.445	3.530
20	3.400	3.446	3.530
40	3.401	3.447	3.531
60	3.402	3.448	3.532
80	3.403	3.449	3.533
100	3.404	3.450	3.533

TABLE 3: DEPENDENCE OF a_c WITH N FOR DIFFERENT VALUES OF ψ AND Da^{-1}=5, Λ=3, λ=5, M_3=3, χ=2.

N	a_c		
	ψ = 1	ψ = 5	ψ = 10
0	3.361	3.445	3.418
20	3.362	3.446	3.419
40	3.363	3.447	3.420
60	3.364	3.448	3.421
80	3.365	3.449	3.422
100	3.366	3.450	3.423

TABLE 4: DEPENDENCE OF a_c WITH N FOR DIFFERENT VALUES OF λ AND FOR Da^{-1}=5, Λ=3, ψ=3, M_3=3, χ=2.

N	a_c		
	λ = 1	λ = 5	λ = 5
0	3.185	3.445	3.753
20	3.185	3.446	3.754
40	3.186	3.447	3.754
60	3.187	3.448	3.755
80	3.188	3.449	3.756
100	3.188	3.450	3.756

TABLE 5: DEPENDENCE OF a_c WITH N FOR DIFFERENT VALUES OF M_3 AND FOR Da^{-1}=5, Λ=3, ψ=5, λ=5, χ=2.

N	a_c		
	M_3 = 1	M_3 = 5	M_3 = 10
0	3.445	3.445	3.445
20	3.446	3.446	3.445
40	3.448	3.446	3.446
60	3.449	3.447	3.446
80	3.450	3.448	3.447
100	3.451	3.449	3.447

TABLE 6: DEPENDENCE OF a_c WITH N FOR DIFFERENT VALUES OF χ AND FOR Da^{-1}=5, Λ=3, ψ=5, λ=5, M_3=3

N	a_c		
	χ = 1	χ = 3	χ = 5
0	3.445	3.445	3.445
20	3.446	3.446	3.446
40	3.447	3.447	3.447
60	3.448	3.448	3.448
80	3.449	3.449	3.449
100	3.450	3.450	3.450

the line z=0. This symmetry of the basic temperature profiles is largely responsible for the stabilizing effect of both ψ and λ.

The variation of R_c with N for different values of Λ, Da^{-1}, ψ, λ, M_3 and χ is exhibited in Figs.3 through 8 respectively. The destabilizing influence of the magnetic parameters N and M_3, and the stabilizing influence of the porous parameters Λ and Da–1 are qualitatively in agreement with the results of the second sound problem. The parameter ψ signifies the temperature in the equilibrium state, while λ is the characteristic of absorption coefficient and distance between the horizontal planes. The stabilizing influence of ψ and λ is evident from Figs.5 and 6. This is due to the fact that radiative transfer tends to damp out any motions which may arise due to the heat transfer from hotter to colder parts of the magnetic fluid. As a result, the effect of thermal radiation is to inhibit Darcy-Brinkman ferroconvection. Noticeably, the destabilizing nature of magnetic forces is not diminished by the effect of Λ and the opposite is true for the parameter λ. From Fig.8, it is evident that the effect of increasing χ is to increase the value of R_c and thus it delays the onset of ferroconvection. However, the stabilizing effect of χ is negligibly small.

Furthermore, we infer from Tables 1 through 6 that convection cell size gets affected by N, Λ, Da^{-1}, Λ, λ and M_3, but insensitive to the variation in χ. In the limiting case of $\psi = N = Da^{-1} = 0$ and $\Lambda = 1$, one obtains the classical values of a_c = 3.117 and and R_c = 1707.76 (Chandrasekhar, 1961). The results of this problem have implications for the utilization of magnetic fluids as heat carriers in the capture of solar energy.

6.0 Conclusions

Darcy-Brinkman instability of a ferrrofluid with the effect of thermal radiation is studied using the technique of small perturbations. The analysis has led to the following conclusions:

- Basic temperature profiles are nonlinear and symmetric with respect to the variations in the radiative parameters ψ and λ. This symmetry is largely responsible for the stabilizing effect of both radiative parameters ψ and λ.
- The destabilizing nature of magnetic forces is not diminished by the effect of ψ and the opposite is true for the parameter λ.
- Nonlinearity of magnetization diminishes the ferroconvection threshold and this effect becomes less strong when M_3 is significantly large.
- The stability of the ferromagnetic fluid increases with an increase in the value of the inverse Darcy number and the Brinkman number.

The study throws light on the effective control of ferroconvection in the presence of porous medium. The results of the study could be exploited to augment or suppress ferroconvection and to corroborate the findings of laboratory based heat transfer experiments involving ferromagnetic fluids.

7.0 References

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