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Ferroconvection in a sparsely distributed porous medium with time-dependent sinusoidal magnetic field

The stability of a horizontal sparsely packed porous layer of a ferromagnetic fluid heated from below is examined when the fluid layer is subjected to time-dependent magnetic field modulation. The effects of the oscillating magnetic field are treated by a perturbation expansion in powers of the amplitude of the applied field. The onset criterion is derived based on the condition that the principle of exchange of stabilities is valid. The stability of the system, characterized by a correction Rayleigh number, is computed as a function of magnetic, porous parameters and the frequency of magnetic field modulation. It is found that the onset of magnetic field modulated ferroconvection can be delayed or advanced by controlling these parameters. The effect of various parameters is found to be significant for moderate values of the frequency of magnetic field modulation. The problem throws some light on external means of regulating convection in ferromagnetic fluid applications.

Keywords: Ferromagnetic fluid, magnetic field modulation, perturbation method, stability, porous medium.

1.0 Introduction

Ferromagnetic fluids (also known as ferrofluids or magnetic fluids) are a form of smart liquid that has been polarised by magnetic forces. Ferromagnetic fluids are made through immersing tiny magnetic nanoparticles in a non-magnetic liquid carrier and encapsulating it inside a molecular solvent to prevent particle coagulation in the presence of an applied external magnetic field. Many experts and industrialists, however, are intrigued by colloidal magnetite, the most carefully researched ferrofluid, owing to its applications ranging in heat transfer, bio-medical and aircraft, to highlight a few. Popplewell (1984), Rosensweig (1997), Berkovsky et al. (1993), Horng et al. (2001).

The utilization of ferroconvection to heat transfer in a layer enclosing ferrofluid is analogous to standard Benard convection and has sparked considerable interest in the literature due to its potential utility as a heat exchanger. Finlayson (1970) first detailed how a horizontal surface containing ferrofluid with fluctuating magnetic susceptibility yields a non-uniform magnetic body force, which results in thermomagnetic convection. This type of thermal movement may be advantageous when classic convection is inadequate, such as in small micro-scale apparatus or in low gravity situations.

Gupta and Gupta (1979) interrogated thermal instability in a magnetic liquid surface with centrifugal acceleration subjected to vertically penetrating magnetic field, proving that overstability is impossible if the Prandtl number is greater than one. Gotoh and Yamada (1982) addressed the linear convective unsteadiness of a magnetic liquid medium warmed from below and sandwiched between two horizontal ferromagnetic blocks. As trial functions, Legendre polynomials are utilised, and the Galerkin method is employed. The magnetization of the boundaries, as well as the inhomogeneity of fluid magnetization, are demonstrated to reduce the critical Rayleigh number, and the effects of magnetization and buoyancy forces are shown to compensate for each other. Stiles et al. (1992) studied the thermoconvective instability of a single surface containing ferromagnetic fluid bound within inflexible horizontal planes at room temperature and driven to a moderate homogeneous externally supplied magnetic field in the vertical position. When heated downward, the roll cells' critical gravitational and magnetic Rayleigh numbers, as well as their critical horizontal wavelength range, correspond with those of Finlayson (1970).

Aniss et al. (2001) investigated the impacts of a time-sinusoidal magnetic field on the onset of convection in a horizontal magnetic layer of fluid heat gained from above and enclosed by isothermal non-magnetic walls. Using a first order Galerkin method, the controlling linear system is reduced to the Mathieu equation with a dissipation factor. As a consequence, the Floquet theory is used to determine the convective threshold in the free-free and rigid-rigid situations. The possibility of generating a conflict between harmonic and

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sub-harmonic modes is addressed at the onset of convection. Kaloni and Lou (2005) calculated theoretical temperature instability of a relatively thin surface of ferrofluid that is heated up from underneath under the action of changing magnetic fields a priori. The eigenvalue of the equation is estimated by using floquet hypothesis, Chebyshev pseudospectral strategy, and QZ technique. For the quasistationary system, both free-free and rigid-rigid boundary conditions are studied, whereas only the rigid-rigid boundary condition is addressed for the model with internal rotation.

In their studies, Engler and Odenbach (2008) Engler and Odenbach (2009) looked at the features of the onset of thermomagnetic convection in ferrofluids being controlled by fixed and frequently varied magnetic fields. In the instance of a static magnetic field, the commencement of convection is determined by the strength of the magnetic field, whereas in the context of a time-modulated magnetic field, an additional reliance on the frequency of magnetic field fluctuation is shown. In general, the experimental results support theoretical predictions about the influence of static and time-modulated magnetic fields on the onset of convection. Matura and Lücke (2009) analyzed how a time-periodic and essentially constant magnetic field affects the linear and non-linear durability of a ferrofluid layer heated from above and below. The floquet theory is being used to define the stability bounds of the static conductive phase for a harmonic and subharmonic output. Complete simulation studies employing the finite difference method were used to produce nonlinear convective conditions. The effects of the lowest and maximum modulation frequencies on the stability boundaries and potential nonlinear oscillations are investigated.

With a vertically applied non-uniform time dependent magnetic field, Bhadauria and Kiran (2014) studied heat transport through an electrically conducting layer of fluid using a weakly nonlinear approach. The Ginzburg-Landau non-autonomous formula is used to determine the thermal expansion coefficient, and Wolfram Mathematica 8 will be used to accomplish it. Suitability was determined using the Runge–Kutta–Fehlberg technique. The Nusselt number is calculated based on a variety of system parameters, with the influence of each parameter on heat flow being highlighted. Heat transfer is facilitated by increasing the magnetic Prandtl number and modulation amplitude. The Chandrasekhar number modulation frequency is employed to maintain the system steady. Magnetic modulation has also been demonstrated to either increase or decrease heat transmission. Kiran et al. (2018) addressed the oscillatory pattern of chaotic magneto-convection in a magnetic force dependent with regard to temporal variations, which would be a follow-up to Bhaduria's research. The conformation and generalizability of the findings are tested by comparing the ND Solve Mathematica/simulink software solution with the RKF45 method, and an acceptable approximation is found. In asynchronous mode, magnetic modulation operates better than in stationary mode. Keshri et al. (2019) used a mathematical technique to predict the temperature profile upon an ionised couple stress liquid with an internal heating under a magnetic force variability. The Prandtl number, internal Rayleigh number, couple stress parameter, and magnetic Prandtl number exert unfavourable impact on the system, meanwhile the Chandrasekhar number has a significant impact. As a consequence, the couple stress parameter and internal Rayleigh number enhance the heat transfer process.

Temperature distribution across fluid-saturated porous materials has sparked a great deal of interest due to its inherent characteristic as well as its increasing application in science and innovation, such as tidal energy resource usage, nuclear waste elimination, building thermal shielding, waste removal in aquifers, solid matrix compact heat exchangers, drying processes, and so on. The discipline's work was headed by Horton and Rogers (1945) and Lapwood (1948), and the entire problem is now known as the Horton-Rogers-Lapwood or Darcy-Benard problem. In this conventional composition, a porous substance is sandwiched between two flat surfaces of similar temperature and heated from below. Several authors, nonetheless, have gotten further depth on the subject, and Nield and Bejan (2006) and Vafai (2015) provide excellent summaries of the growing corpus of research in the field.

In the instance of a time-dependent buoyancy force caused by gravity modulation, the stability of a horizontal fluid and fluid-saturated porous layer heated from below is investigated by Malashetty and Padmavathi (1997). The gravity modulation has a considerable impact on the system's stability limitations, according to a linear stability study. As a function of modulation frequency, Prandtl number, and porosity parameter, the change in the critical Rayleigh number is computed. The low frequency g-jitter has been discovered to have a major impact on the system's stability. As degenerate examples of the Brinkman model, the Darcy limit and viscous flow limit are derived. Govender (2004) used the linear stability theory to examine the effects of gravity modulation on convection in a homogeneous porous layer heated from below analytically. The gravitational field has a constant component and a sinusoidally changing component, which is analogous to a vertically oscillating porous layer exposed to constant gravity. The linear stability findings are provided for the situation of low amplitude vibration, in which it is demonstrated that raising the vibration frequency stabilises the convection.

Malashetty and Basavaraja (2004) used a linear stability analysis to investigate the influence of time-periodic boundary temperatures on the beginning of double diffusive convection in a fluid-saturated anisotropic porous media. The critical values of the thermal Rayleigh number and wave number are computed using a perturbation approach based on a tiny amplitude of the applied temperature modulation. As a function of frequency of modulation, viscosity ratio, anisotropy parameter, porous parameter, Prandtl number, diffusivity ratio, and solute Rayleigh number, the correction thermal Rayleigh number is computed. At intermediate frequencies, the influence of several physical characteristics is shown to be considerable. They discovered that by fine-tuning the frequency of wall temperature modulation, they can accelerate or postpone the beginning of double diffusive convection. The impact of various parameters on the system's stability is examined.

Recently, Mahajan and Parashar (2020) investigated the linear and weakly nonlinear instability in a rotating anisotropic magnetic fluid layer, When the layer is internally heated and the solid matrix and fluid are not in local thermal equilibrium. To investigate the transient behaviour of the Nusselt number at the lower boundary, the Runge–Kutta–Gill numerical technique is employed to solve the finite-amplitude problem. The Taylor number and thermal anisotropy parameter were discovered to have a stabilising impact on convection. Heat transfer decreases as the Taylor number and thermal anisotropy parameter rise.

The topic of convection control is relevant and interesting in a wide range of ferromagnetic fluid applications, and it is also theoretically hard. The unmodulated Rayleigh-Bénard issue of convection in a ferromagnetic fluid has received a lot of attention. However, significant attention has been devoted to the combined effect of magnetic field modulation and sparsely packed porous medium on the onset of ferroconvection in a horizontal layer. We want to give a fundamental knowledge of the function of harmonic vertical vibrations, porous parameter, and magnetic factors in controlling natural convection.

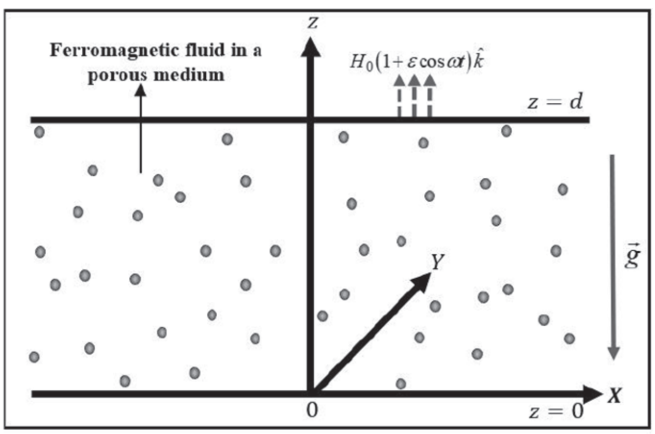

Fig.1. Schematic diagram

2.0 Formulation of the problem

Consider a ferromagnetic fluid layer placed in the middle of two horizontal infinite planes positioned at z = 0 and z = d with gravity, $\vec{g} = -g\hat{k}$ acts towards down, g is the acceleration due to gravity. The upper and lower surfaces are retained at uniform temperature gradient ΔT. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The Boussinesq approximation is assumed to investigate the density variation in the fluid layer.

$$\nabla \cdot \vec{q} = 0 \qquad \text{... (1)}$$

$$\rho_0\left[\frac{1}{\varepsilon_p}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon_p^2}\left(\vec{q}\cdot\nabla\right)\vec{q}\right] = -\nabla p + \rho\vec{g} - \frac{\mu_f}{K}\vec{q} + \bar{\mu}_f\nabla^2\vec{q} + \nabla\cdot\left(\vec{H}\vec{B}\right) \qquad \text{... (2)}$$

$$\varepsilon_p C_1\left[\frac{\partial T}{\partial t} + \left(\vec{q}\cdot\nabla\right)T\right] + \mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\cdot\left[\frac{\partial \vec{H}}{\partial t} + \left(\vec{q}\cdot\nabla\right)\vec{H}\right] + \left(1-\varepsilon_p\right)\left(\rho_0 C\right)_s\frac{\partial T}{\partial t} = K_1\nabla^2 T \qquad \text{... (3)}$$

$$\rho = \rho_0\left[1 - \alpha\left(T - T_a\right)\right] \qquad \text{... (4)}$$

$$\vec{M} = \frac{\vec{H}}{H}M\left(H, T\right) \qquad \text{... (5)}$$

$$M = M_0 + \chi_m\left(H - H_0\right) - K_m\left(T - T_a\right) \qquad \text{... (6)}$$

Various physical quantities appearing in Eqs. (1) through (6) have their usual meaning Finlayson (1970), Thomas and Maruthamanikandan (2018).

The relevant maxwell equations are

$$\nabla\cdot\vec{B} = 0, \quad \nabla\times\vec{H} = 0, \quad \vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) \qquad \text{... (7)}$$

where $\vec{q}$ is the velocity of fluid, ρ the density, ρ_0 a reference density, ε_p the porosity, p the pressure, μ_f the dynamic viscosity, $\bar{\mu}_f$ is the effective viscosity, μ_0 the magnetic permeability, T the temperature, $\vec{H}$ the total magnetic field, $\vec{M}$ the magnetization, $\vec{B}$ the magnetic induction, K_1 the thermal conductivity, α the coefficient of thermal expansion, T_a a reference temperature, $C_1 = \rho_0 C_{V,H} - \mu_0\vec{H}\cdot\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}$, where $\rho_0 C_{V,H}$ the specific heat at constant volume and magnetic field, χ_m is the differential magnetic susceptibility and K_m is the pyromagnetic coefficient. The lower and upper surface temperatures respectively are $T = T_a + \frac{1}{2}\Delta T$ at z = 0 and $T = T_a - \frac{1}{2}\Delta T$ at $z = d$.

The external magnetic force is modulated harmonically in time by varying the magnetic field acting vertically upward.

$$\vec{H}_0^{ext}\left(t\right) = H_0^{ext}\left(t\right) = H_0\left(1 + \varepsilon\cos\omega t\right)\hat{k}$$

where H_0 is the uniform magnetic field, ε is the small amplitude, ω is the frequency and t is the time.

3.0 Basic state

The quiescent basic state is represented by

$$\left.\begin{aligned}&\vec{q}=\vec{q}_b=(0,0,0),\ p=p_b(z),\ \rho=\rho_b(z),\\&T=T_b(z),\vec{M}=\vec{M}_b=(0,0,M_0(z)),\\&\vec{H}=\vec{H}_b=(0,0,H_0(z,t))=H_0^{ext}(t)\hat{k},\\&\vec{B}=\vec{B}_b=(0,0,B_0(z))\end{aligned}\right\}\qquad \text{... (8)}$$

In the basic state, the pressure, the temperature, the magnetic field, magnetic induction and magnetization equations are as follows

$$\left.\begin{aligned}&-\frac{\partial p_b}{\partial z}-\rho_b g+B_0\frac{\partial H_b}{\partial z}=0,\\&T_b=T_a+\Delta T\beta,\\&\rho_b=\rho_0(1-\alpha\Delta T\beta),\\&H_b=[H_0+\eta\Delta T\beta]\psi,\\&M_b=[M_0+\eta\Delta T\beta]\psi,\\&B_b=\mu_0(M_0+H_0)\end{aligned}\right\}\qquad \text{... (9)}$$

with $\psi=\dfrac{(1+\varepsilon\cos\omega t)}{(1+\chi_0)}$, $\eta=\dfrac{H_0\chi_0}{(1+\chi_0)T_a}$, and $\beta=\left(\dfrac{1}{2}-\dfrac{z}{d}\right)$

4.0 Linear stability analysis

The perturbation technique is used to examine the stability of the basic state, we superpose immeasurably small perturbations on the basic state of the form

$$\left.\begin{aligned}&\vec{q}=\vec{q}_b+\vec{q}',p=p_b+p',\rho=\rho_b+\rho',\\&T=T_b+T',\vec{H}=\vec{H}_b+\vec{H}',\\&\vec{M}=\vec{M}_b+\vec{M}',\vec{B}=\vec{B}_b+\vec{B}'\end{aligned}\right\}\qquad \text{... (10)}$$

where primes represent perturbed quantities. Substituting Eq. (10) into Eqs. (1) – (7) and using basic state solution, we obtain the following equations

$$\nabla\cdot\vec{q}'=0 \qquad \text{... (11)}$$

$$\rho'=-\alpha\rho_0 T' \qquad \text{... (12)}$$

$$\begin{aligned}\frac{\rho_0}{\varepsilon_p}\left[\frac{\partial\vec{q}'}{\partial t}\right]&=-\nabla p'+\alpha\rho_0 gT'\hat{k}+\bar{\mu}_f\nabla^2\vec{q}'\\&-\frac{\mu_f}{K}\vec{q}'+\mu_0(M_0+H_0)\frac{\partial\vec{H}'}{\partial t}\\&-\left(\frac{\mu_0\chi_0H_0(1+\varepsilon f)\Delta T}{T_a(1+\chi_0)d}\right)\frac{\partial\phi'}{\partial z}\hat{k}\\&+\left(\frac{\mu_0\chi_0^2H_0^2(1+\varepsilon f)^2\Delta T}{T_a^2(1+\chi_0)^3d}\right)T'\hat{k}\end{aligned}\qquad \text{... (13)}$$

$$\begin{aligned}&C_3\frac{\partial T'}{\partial t}-\frac{\varepsilon_pC_2\Delta T}{d}w'-\mu_0\chi_0H_0\psi\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial z}\right)\\&+\frac{\mu_0\chi_0^2H_0^2\Delta T}{T_ad(1+\chi_0)}\psi^2w'-\mu_0\chi_0\left(\frac{\partial\phi'}{\partial z}\right)\frac{\partial}{\partial t}H_0\psi\\&-\frac{\mu_0\chi_0H_0}{T_a(1+\chi_0)}\psi T'\frac{\partial}{\partial t}H_0(1+\varepsilon f)\\&+\frac{\varepsilon_p\mu_0\chi_0H_0^2}{T_a}\psi^2\left(\frac{\partial T'}{\partial t}-w'\frac{\Delta T}{d}\right)=K_1\nabla^2T'\end{aligned}\qquad \text{... (14)}$$

$$(1+\chi_0)\nabla^2\phi'-\left(\frac{\chi_0H_0\psi}{T_a}\right)\frac{\partial T'}{\partial z}=0 \qquad \text{... (15)}$$

where $C_3=\varepsilon_pC_2+(1-\varepsilon_p)(\rho_0C)_s$, $\vec{H}=\nabla\phi'$, $C_2=\rho_0C_{V,H}$, $\vec{q}'=(u',v',w')$, with ϕ' being the magnetic potential. In Eq. (13) pressure term can be eliminated by applying curl twice on it and then render the resulting equation and Eqs. (13) – (15) dimensionless over the following transformations $(x^*,y^*,z^*)=\left(\dfrac{x}{d},\dfrac{y}{d},\dfrac{z}{d}\right)$,

$$\phi^*=\frac{\phi'}{\left(\dfrac{H_0\chi_0\psi\Delta Td}{T_a(1+\chi_0)}\right)}\quad t^*=\frac{t}{\left(\dfrac{C_2d^2}{K_1}\right)},$$

$$W^*=\frac{w'}{\left(\dfrac{K_1}{C_2d}\right)},\quad T^*=\frac{T'}{\Delta T},\ \text{and}\quad \omega^*=\frac{\omega}{\left(\dfrac{K_1}{C_2d^2}\right)}$$

to obtain (after ignoring the asterisks)

$$\begin{aligned}&\left(\frac{1}{P_r}\frac{\partial}{\partial t}+D_a^2-\Lambda\nabla^2\right)\nabla^2W\\&\qquad=\left[R+RM_1(1+\varepsilon f)^2\right]\nabla_1^2T\\&\qquad\quad-RM_1(1+\varepsilon f)^2\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right)\end{aligned}\qquad \text{... (16)}$$

$$\begin{aligned}&\lambda_p\frac{\partial T}{\partial t}-W+\frac{M_2}{\varepsilon_p}\psi^2W\\&-M_2\frac{1}{H_0}\frac{\partial}{\partial t}H_0\psi\left(\frac{\partial\phi}{\partial z}\right)\\&+\frac{M_2}{\chi_0}\left(\psi(1+\varepsilon f)\right)\left(\frac{\partial T}{\partial t}-W\right)\\&-M_2\left(\psi(1+\varepsilon f)\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)\\&-M_2\frac{1}{H_0}T\frac{\partial}{\partial t}H_0(1+\varepsilon f)=\nabla^2T\end{aligned}\qquad \text{... (17)}$$

$$\left(\frac{\partial^2}{\partial z^2}+\nabla_1^2\right)\phi=\frac{\partial T}{\partial z} \qquad \text{... (18)}$$

where $\lambda_p = \dfrac{C_3}{\varepsilon_p C_2}$, ω, ω is the frequency of modulation, $\nabla_1^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \dfrac{\partial^2}{\partial z^2}$.

The dimensionless parameters Kaloni and Lou (2005), Thomas and Maruthamanikandan (2018) are P_r the Darcy-Prandtl number, R the Darcy-Rayleigh number, M_1 the buoyancy-magnetization parameter, M_2 the magnetization parameter, RM_1 the magnetic Rayleigh number, D_a the porous parameter and Λ the Brinkman number.

The parameter M_2 is equivalent to the order of 10^{-10} Finlayson (1970). Hence M_2 can be omitted in further calculations. The suitable boundary conditions are Malashetty and Padmavathi (1997).

$$W = \frac{\partial^2 W}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0, 1 \text{ at } z = 0,\ 1 \qquad \text{... (19)}$$

It is suitable to state the whole problem in terms of the vertical component of the velocity W. Upon combining Eqs. (16)–(18), we obtain the following equation.

$$L_1 L_2 L_3 \nabla^2 W = R L_3 \nabla_1^2 W + RM_1 \left(1 + \varepsilon f\right)^2 \nabla_1^4 W$$

where, $L_1 = \left(\dfrac{1}{P_r}\dfrac{\partial}{\partial t} + D_a^2 - \Lambda \nabla^2\right)$, $L_2 = \left(\dfrac{\partial}{\partial t} - \nabla^2\right)$ and $L_3 = \left(\dfrac{\partial^2}{\partial z^2} + \nabla_1^2\right)$

where $f = \mathrm{Re}\left\{e^{-i\omega t}\right\} = \cos \omega t$. The boundary conditions in Eq.(19) can also be expressed in terms of W in the form Chandrasekhar (1961).

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = 0 \text{ at } z = 0, 1 \text{ at } z = 0,\ 1 \qquad \text{... (21)}$$

5.0 Method of solution

The eigenfunctions, W and the eigenvalues, R associated with the above eigenvalue problem for a modulated magnetic field that is different from the constant magnetic field by a small quantity of order ε. We therefore assume the solution of Eq. (20) in the form Veneziant (1969).

$$\begin{pmatrix} W \\ R \end{pmatrix} = \begin{pmatrix} W_0 \\ R_0 \end{pmatrix} + \varepsilon \begin{pmatrix} W_1 \\ R_1 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} W_2 \\ R_2 \end{pmatrix} + \qquad \text{... (22)}$$

where is the critical Darcy-Rayleigh number for the corresponding unmodulated problem. The expression for the Rayleigh number R_0 is given by

$$R_0 = \frac{\left[D_a^2 + \Lambda\left(\pi^2 + \alpha^2\right)\right]\left(\pi^2 + \alpha^2\right)^3}{\alpha^2\left[\pi^2 + \left(1 + M_1\right)\alpha^2\right]} \qquad \text{... (23)}$$

where $\alpha^2 = \alpha_x^2 + \alpha_y^2$ being overall horizontal wavenumber α_x and α_x being wavenumbers in x and y directions respectively.

Following the analysis of Malashetty and Padmavathi Malashetty and Padmavathi (1997), one obtains the following expression for R_2 (the first non-zero correction to R_0)

$$R_2 = \frac{2R_0^2 M_1^2 \alpha^6}{\left[\pi^2 + \left(1 + M_1\right)\alpha^2\right]} \sum_{n=1}^{\infty} \frac{E_n}{F_n} \qquad \text{... (24)}$$

where

$$E_n = \omega^2 \frac{\lambda_p}{P_r}\left(n^2\pi^2 + \alpha^2\right)^2 - D_a^2\left(n^2\pi^2 + \alpha^2\right)^3 + R_0\alpha^2\left[n^2\pi^2 + \left(1 + M_1\right)\alpha^2\right] - \Lambda\left(n^2\pi^2 + \alpha^2\right)^4$$

with

$$A_1 = \omega^2 \frac{\lambda_p}{P_r}\left(n^2\pi^2 + \alpha^2\right)^2 - D_a^2\left(n^2\pi^2 + \alpha^2\right)^3 + R_0\alpha^2\left[n^2\pi^2 + \left(1 + M_1\right)\alpha^2\right] - \Lambda\left(n^2\pi^2 + \alpha^2\right)^4$$

and

$$A_2 = \frac{\omega}{P_r}\left(n^2\pi^2 + \alpha^2\right)^3 + \lambda_p \Lambda \omega\left(n^2\pi^2 + \alpha^2\right)^3 + \lambda_p D_a^2 \omega\left(n^2\pi^2 + \alpha^2\right)^2$$

The Rayleigh number R at its critical value is calculated upto $O(\varepsilon)^2$ by computing R_0 and R_2 at $\alpha_0 = \alpha_c$, Thomas and Maruthamanikandan (2013) where α_c is the value at which R_0 is minimum. Supercritical instability occurs provided R_{2c} is positive. On the other hand, subcritical instability is said to occur when R_{2c} turns out to be negative.

6.0 Results and discussion

An analytical study has been undertaken on the influence of time-periodic magnetic field fluctuation on the onset of ferromagnetic convection in a horizontal porous layer. The corrected Darcy-Rayleigh number R_{2c} is found to be proportional to the modulation frequency ω, magnetic parameter M_1, Prandtl number P_r, porous parameter D_a, and Brinkman number Λ, providing small amplitude variation, and the regular perturbation technique would be used. Figs.2 to 5 shows the impact of these factors on the system's stability.

At minimal values of ω, the usual view is that R_{2c} is negligible. This reveals that the modulated magnetic field seems to have a destabilising impact. R_{2c} becomes significant over moderate and large values of ω and so convection is delayed.

The influence of the buoyant magnetization parameter is discussed in Fig.2. R_{2c} clearly grows as M_1 increases, as long as ω is minimal. If ω is moderate and large, however, the pattern reverses. In addition, the magnetic mechanism reduces the impact of magnetic field variation. Furthermore, when M_1 is large enough, the insignificant effect of M_1 on stability may be shown.

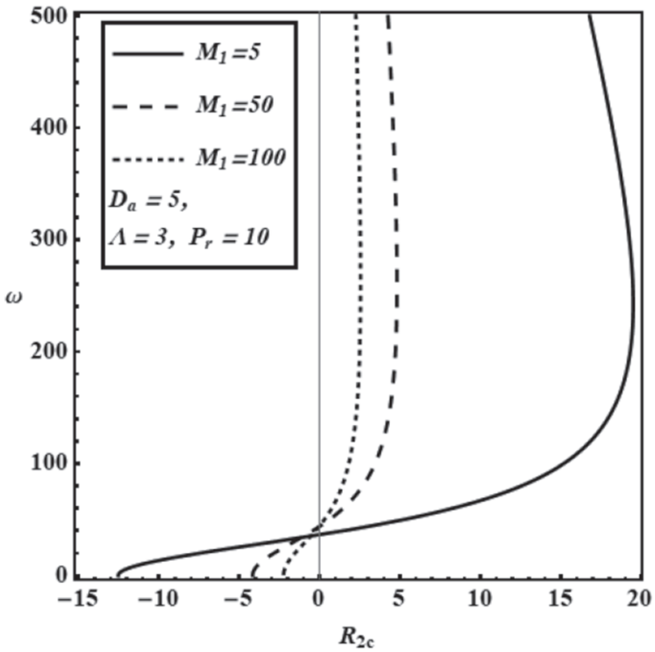

Fig.2. Variation R_{2c} of with respect to ω and M_1

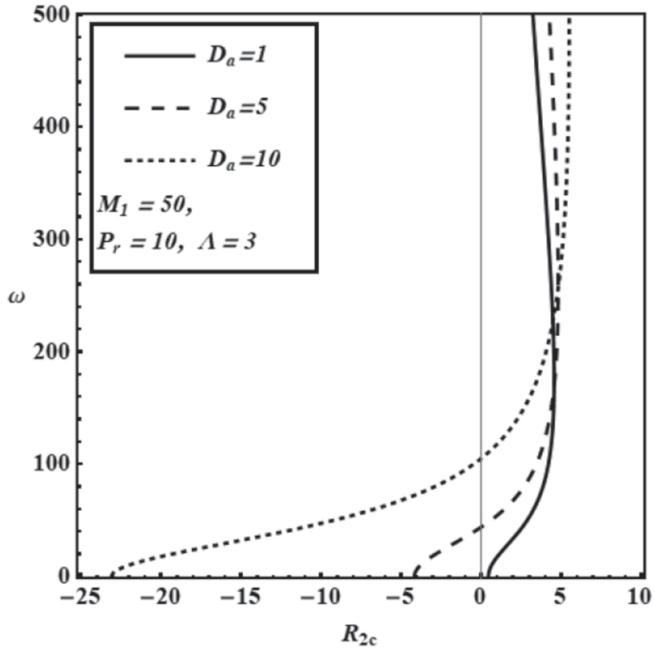

Fig.4: Variation of R_{2c} with respect to ω and D_a

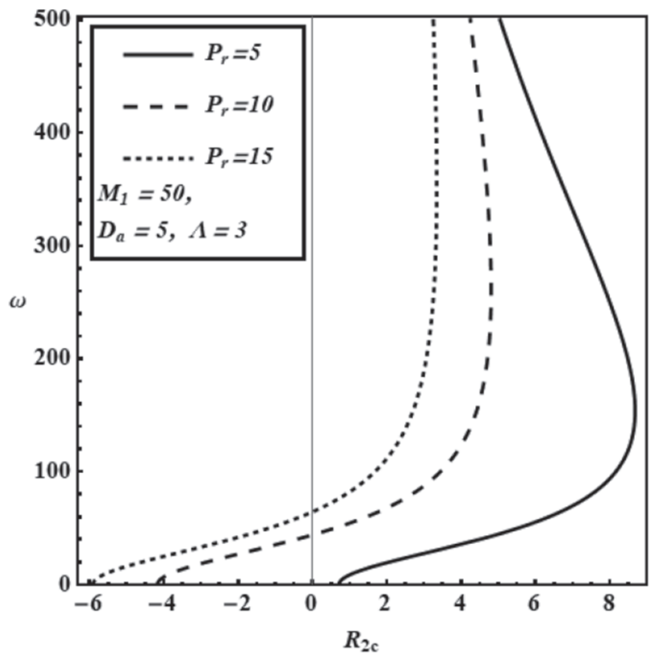

Fig.3: Variation of R_{2c} with respect to ω and P_r

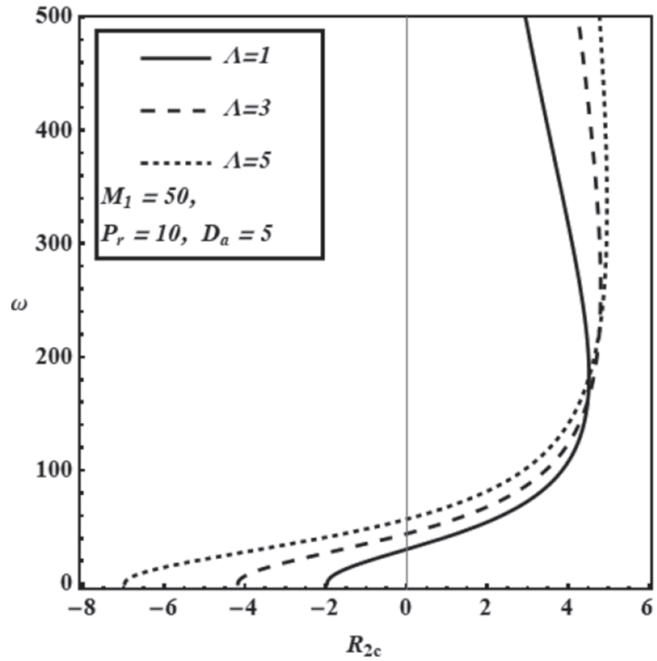

Fig.5: Variation of $R_{2c}$with respect to ω and Λ

Fig.3 outlines the impact of the Prandtl number P_r on R_{2c} with regard to ω. The value of R_{2c} is reduced as incrementing the value of P_r, provided ω is small. Also, R_{2c} shows same effect, for moderate and large value of ω.

Figs.4 and 5 resemble the deviance in D_a and Λ over the critical correction Darcy-Rayleigh number, respectively. When the values of D_a and Λ are raised, R_{2c} falls, showing that both the factors have a convective influence on ferromagnetic convection. However, if ω is small enough, the Darcy number D_a and Brinkman number Λ shows destabilizing effect on the system. Equivalently D_a and Λ has a stabilizing effect when ω is large enough.

7. Conclusions

The effect of magnetic field modulation on ferromagnetic porous medium convection is studied using the procedure of regular perturbation. The investigation has led to the following conclusions:

- Subcritical instability manifests by virtue of modulated magnetic field for low frequency.
- Magnetic field modulation and the magnetic mechanism have mutually opposed effect on the system provided the magnetic field modulation frequency is small as well as moderate.
- Prandtl number enhances the amplifying effect of magnetic field modulation irrespective of the range of frequency.
- Effects of magnetic force, porous medium and magnetic field modulation disappear when the frequency of the time-periodic magnetic force is considerably large.

In conclusion, the threshold of ferromagnetic porous medium convection could be hastened or delayed through magnetic field modulation by tuning the frequency of magnetic field modulation. Hence the magnetic field modulation and porous mechanisms could be exploited to straighten out issues arising in situations involving convective instability of ferromagnetic fluids.

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