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A characterization method of rock fracturing brittleness based on multiple linear regression

The brittle characteristics of rock mass is the key to the evaluation of unconventional oil and gas reservoir transformation effect. In this paper, in order to accurately brittle rock mass, the multifractal description method of multi-scale fracture distribution was established by means of CT scanning imaging. Considering the multi fractal characteristics of fracture in rock mass, a new method based on multiple linear regression was established to characterize shale brittleness. The B layer of A block in Songliao Basin, China, is selected as the research object, and the evaluation model of the brittle characteristics of rock mass fracture is obtained. The results show that the new method of brittle characterization is in good agreement with the actual results, which lays a foundation for the subsequent research on unconventional oil pressure cracking.

Key words: *Unconventional oil and gas, rock fracturing,brittleness index, multiple linear regression, multifractal.*

1. Introduction

The brittleness of rock is closely related to the transformation of oil and gas reservoirs. The brittleness index is an important index to evaluate the mechanical properties of the reservoir, wellbore stability and hydraulic fracturing effect. However, experts in various fields are different from each other, and there is not a unified and standard definition of brittleness. A. Morley [1] and M. Hetenyi [2] define brittleness as the absence of material plasticity; J. G. Ramsey [3] believes that when the cohesion of the rock is lost, the material is brittle failure; L. Obert and W. I. Duvall [4] define brittleness as: brittle properties similar to iron and most of the rock material reach or slightly exceed the yield strength and failure. At present, in rock mechanics, geology and related disciplines scholars believe that minimal fracture or damage before the show or no plastic deformation characteristics of rock brittle means. However, the nature of brittle material has already reached consensus on the following: (1) the damage occurred in low strain; (2) the brittle failure form by the internal micro crack control; (3) rock by fine particulate material composition; (4) the high compressive strength and tensile strength ratio (compression ratio); (5) high rebound power (6); large internal friction angle; (7) crack development completely when the hardness is tested [5].

The properties of brittle rocks are closely related to their mechanical properties. Many scholars have done a lot of research work in this area, and formed a number of evaluation methods. However, most of the brittle indexes are analyzed by single factor analysis. According to the study of rock brittleness in recent years, the single factor cannot be used to evaluate the brittleness of rock. However, the results of the calculation of brittle index are not the same: the stronger the brittle rock, the stronger the residual energy, the faster the elastic energy release. In this paper, using multiple linear regression analysis of rock elastic modulus and Poisson ratio and mineral composition between quantitative characterizations, combined with elastic parameter method and rock mineral analysis, and considering the multifractal distribution characteristics of the fracture rock mass, and put forward a new kind of rock brittleness evaluation method based on multiple linear regression method. The actual calculation results show that the new method in this paper is consistent with the actual measurement results.

2. Methods for characterizing rock brittleness

2.1. Rock brittleness definition

Rock is formed in a certain geological environment, it has a certain structural and structural characteristic of mineral composition. The rock structure refers to the degree of crystallization of minerals in the rock, mineral or rock particles in shape and size and their mutual connection; rock structure refers to the appearance of the rock, the rock appearance is through the minerals in the rock or rock debris

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particles may have in the space arrangement and in rocks void distribution and filling are shown.

Rock brittleness is the comprehensive characteristic of material, and is a kind of state quantity of rock. The definition is: (1) Brittleness is the comprehensive characteristics of rock. The results show that the rock is characterized by the brittleness of the internal structure of the rock and external loading conditions of the characteristics of the common features. (2) Brittleness is a state of rock. It is shown that the rock brittleness is a dynamic changing. The domestic and foreign research scholars from different views on the rock brittleness evaluation index of indoor test mainly come from the following 4 evaluations: based on the brittle mineral composition, based on the strength of the brittle, based on the appearance, based on the stress and stain curve and based on energy dissipation.

2.2. Brittleness evaluation based on mineral composition

Based on the analysis of the mineral composition, the brittleness prediction model of rock is established by analyzing the difference of the mechanical properties of different mineral compositions. At present, the quartz, calcite and clay minerals have the greatest influence on shale brittleness. Quartz is one of the most important brittle minerals. The rock brittleness is analyzed by rock mineral composition triangulation, according to mineral composition of rocks in the triangle of punctuation, then according to the distribution characteristics of test rock in the triangle, with the macro characteristics of rock. The specific method of calculating brittleness can be expressed as:

$$BRIT_1 = \frac{V_{quartz}}{V_{quartz} + V_{carbonatite} + V_{clay}} \times 100 \quad \text{...} \quad (1)$$

Where V indicates the volume content of different mineral types; $BRIT_1$ represents index of rock brittleness.

Diao, et al. [6] refined formula (1) and got new formula (2) for calculating the brittleness of mineral content:

$$BRIT_2 = \frac{V_{quartz} \times (E_{quartz}/\nu_{quartz})}{V_{quartz} \times (E_{quartz}/\nu_{quartz}) + V_{calcite} \times (E_{calcite}/\nu_{calcite}) + V_{clay} \times (E_{clay}/\nu_{clay})} \times 100 \quad \text{...} \quad (2)$$

In this case, the brittleness evaluation index is in line with the calculation results of other brittle indexes. The larger the brittleness index is, the larger is the rock brittleness. And the brittle characteristic profile of the whole well can be quickly obtained by the correction of the mineral composition.

However, it is easy to neglect the influence of digenesis and formation pressure on rock brittleness. In fact, the digenesis is very brittle to the rock, and the rock with the same mineral composition has different digenetic processes. According to the present study, the greater the confining pressure is, the smaller is the rock brittleness. The calculation formula does not take into account digenesis and confining pressure.

2.3. Brittleness evaluation method based on strength

Based on the strength of the brittleness evaluation method is through the test results of uniaxial compressive strength and splitting tensile strength test of rock strength parameters, the uniaxial compressive test and split test is simple and convenient, easy to test, therefore, widely used by the rock strength parameters based on brittleness index. The basic idea for the use of compressive and tensile strength of the differences in the evaluation of brittleness, such as R. Altindag [7] (formula (3)) and S. Kahramana [8] (formula (4)).

$$B_1 = \sigma_c / \sigma_t \quad \text{...} \quad (3)$$

$$B_1 = \sigma_c \sigma_t / 2 \quad \text{...} \quad (4)$$

The compressive strength and tensile strength parameters were obtained through laboratory tests. According to the macroscopic characteristics of rock, the rock brittleness grade is divided, as shown in Fig.1.

From Fig.1 we can see that the larger the pressure of the rock is, the greater is the compressive strength of rock, the larger the tensile strength of rock is, the more likely it is that there will be the same pressure of different brittle rocks. Therefore, the tensile strength of pressure ratio to some extent can qualitatively reflect the rock brittle, but in quantity insensitive, rating method based on the strength of brittle rock affected by ignoring the stress on rock strength, cannot be used as the evaluation of complex characteristics of brittle rock under stress.

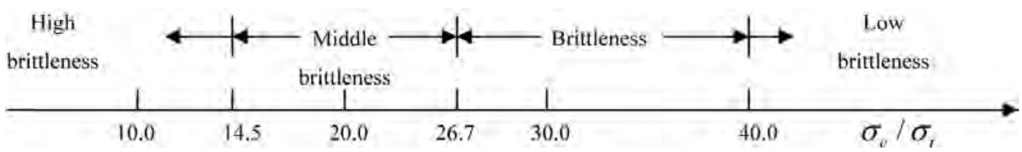

Fig.1 Rock brittleness represented by splitting tensile strength

2.4. Brittleness evaluation method based on representation

Based on the representation of the characteristics of the brittleness of the evaluation method is based on the characteristics of the failure of the material to evaluate the brittleness. It includes brittleness evaluation method based on hardness, brittleness evaluation method based on robustness and brittleness index based on debris content Lawn, B. R. et al [9]. Based on the hardness of the ceramic material, the brittleness evaluation index shows that the material is easy to deform and not easy to break. Yagiz [10] based on the brittleness of the evaluation index, indicating that the deeper the penetration depth, the greater is the brittleness. Protodyakonov [11] presents a brittle index based on the detrital content, which indicates that the percentage of the content of the material is less than that of a certain size. Most of these methods are based on the evaluation of the macroscopic characteristics of the materials, and there is no theoretical basis for the definition of evaluation index. At the same time, the macroscopic characterization of rock is influenced by environmental factors and experimental conditions, it has a large randomness, and the test can be replicated, and there is no perfect theoretical support.

2.5. Brittleness evaluation method based on total stress strain

Based on the full stress strain method, the rock parameters are obtained according to the uniaxial and three axial test chambers. Single axis and three axis test to simulate rock in good condition, mechanical characteristics curves can reflect the rock in the role of internal and external factors, not only can judge rock brittleness qualitatively, but also quantitatively reflect the brittle characteristics. Therefore, the use of stress and strain curves to evaluate brittleness is a common indoor research method. In this paper, the brittleness of rock is evaluated based on the stress-strain curves.

Scholars at home and abroad based on the stress-strain curve have established a number of different indicators, this paper focuses on Li Qinghui [12] (2012), Zhou Hui [13] (2014), Boris Tarasov [14] (2013) who put forward the brittleness evaluation index.

The brittleness evaluation index proposed by Li Qinghui reflects the difficulty of brittle failure with the peak strain, and the shape of the peak curve indicates the brittleness. Formula for:

$$CS_{BRIT} = \frac{\sigma_p - \sigma_r}{E_p\left(\varepsilon_r - \varepsilon_p\right)} \quad \dots \quad (5)$$

$$B = \frac{\varepsilon_p - \varepsilon_n}{\varepsilon_m - \varepsilon_n} + \alpha CS_{BRIT} + \beta CS_{BRIT} + \eta \quad \dots \quad (6)$$

where ε_p is the peak strain, ε_m, ε_n are the maximum and minimum peak strain of the same rock, σ_p, σ_r are the peak stress and residual stress, ε_p, ε_r are the peak strain and residual strain. E_p is the tangent modulus at the peak, *a, b,* η are the adjustment parameters.

According to this formula, if the curve after the peak of the same, with the increase of the peak strain increases the brittleness, and the peak strain is small, the bigger brittleness of contradiction, and α, β, η require a lot of experimental fitting, and the fitting results due to different lithology, different block parameters, calculation results are also different. It cannot be used as a general evaluation index, and cannot establish a unified evaluation criteria.

Schematic diagram of composite brittle parameter value is shown in Fig.2:

Zhou puts forward the evaluation of the relative magnitude and absolute rate of post peak stress drop

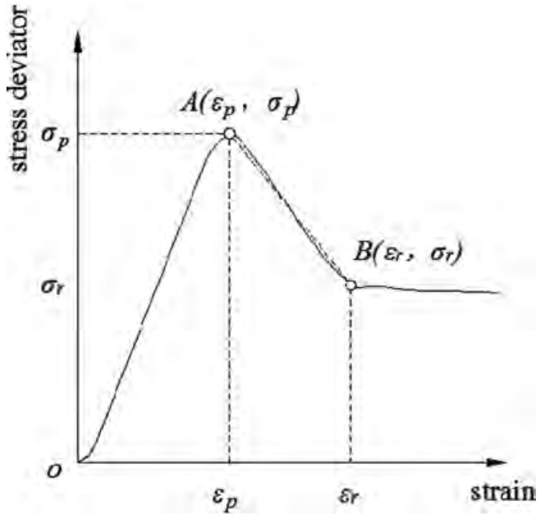

Fig.2 Schematic diagram of composite brittle parameter value

formula for:

$$B = \frac{\tau_p - \tau_r}{\tau_p} \frac{\lg\left|k_{ac\ (AC)}\right|}{10} \quad \dots \quad (7)$$

where τ_p and τ_r are peak stress and residual stress respectively, $k_{ac(AC)}$ is the absolute value of the slope after the peak curve.

Schematic diagram is shown in Fig. 3:

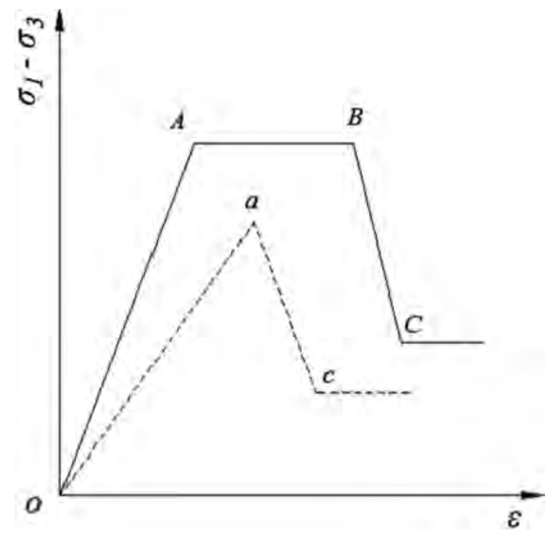

Fig.3 Schematic diagram of the brittleness index

The method is used to evaluate the brittleness of rock after the post peak feature, that is to say, it can be used to describe the post peak brittleness. But ignore the pre peak rock characteristics, and the calculation formula in the calculation of the slope of the curve when the queen, if the rock is in pure brittle, $k_{ac(AC)} = 0$, $\lg\left|k_{ac(AC)}\right|$ are unsolvable, super brittle rock, this method cannot effectively carry out the evaluation.

Boris Tarasov proposed a method for evaluating brittleness based on energy balance relation before peak to peak. Methods two kinds of stress-strain curves were obtained. Formula for:

Brittleness evaluation method of the first kind of characteristic curve

$$B=\frac{E}{M} \quad \text{... (8)}$$

Brittleness evaluation method of second kinds of characteristic curves

$$B=\frac{M-E}{M} \quad \text{... (9)}$$

where E is the elastic modulus of rock, and M is the slope of the peak of the rock.

The brittleness evaluation criterion is that the bigger the brittleness index is, the stronger is the rock brittleness. When the brittleness index is 0, the rock is pure, the brittleness index is greater than 0, and the brittleness is super brittle. From the above evaluation methods, can be seen at present brittleness, brittleness evaluation method is still in the rock crushing features of macro, essentially still cannot explain the formation and evolution of rock brittle.

3. Multifractal characterization of fracture characteristics of shale

3.1. Multifractal method

Assuming R^d is a D dimensional Euclidean space or metric space, is a subset of R^{d}'s D, which is a measure of the support of the μ. If (F, μ) fractal sets can be represented as a union of several fractal subsets, and each subset of fractal have different fractal dimensions, (F, μ) is called multifractal.

If the research object (F, μ) is divided into N units with the largest scale e of S_i (i = 1,2,..., N), r_i is S_i of the linear size, P_i is the measure of S_i (such as probability or quality), and $\sum_{i=1}^{N} P_i = 1$. For different units, P_i may be different, and it can be characterized by different scaling exponents α_i, that is:

$$P_i \propto r_i^{\alpha_i} \quad i=1,2,\cdots,N \quad \text{... (10)}$$

where α_i is singularity index. If the line size tends to be zero, the upper form can be represented as:

$$\alpha_i = \lim_{r_i \to 0} \frac{\ln P_i}{\ln r_i} \quad \text{... (11)}$$

The physical meaning of α_i is the size of the region.

Defined cell number $N_a(e)$ as shown in formula (12)

$$N_\alpha(e) \propto e^{-f(\alpha)}, e \to \quad \text{... (12)}$$

Available

$$f(\alpha) = -\lim_{e \to 0} \frac{\ln N_\alpha(e)}{\ln e} \quad \text{... (13)}$$

The physical meaning of $f(\alpha)$ is the fractal dimension with the same value α is called multifractal spectrum. The interior of a complex fractal can be divided into a series of subsets of different values. α □ $f(\alpha)$ is the basic relation of the local feature of multifractal.

$f(\alpha)$ is called multifractal spectrum to characterize the fractal features of subsets; If the research object is a single fractal, $f(\alpha)$ is a certain value. $f(\alpha)$ generally showed unimodal distribution.

Import the statistical moment function:

$$M(e,q) = \sum_{i=1}^{N} P_i^q(e) \quad \text{... (14)}$$

$q \in (-\infty, \infty)$, which is called the statistical moment order, is mainly used to characterize the non-uniform degree of multifractal. $M(e, q)$ represents the size of the function $P_i(e)$ value. Assuming $P_m(e)>>P_j(e)$, when $q>>1$, apparently $P_i^q(e)$ plays a major role in $\sum_{i=1}^{N} P_i^q(e)$, $M(e, q)$ reflects the nature of the high probability region. On the contrary, when $q<<-1$, $M(e, q)$ reflects the nature of the small probability distribution region.

For a given order q, function $\tau(q)$ satisfied $M(e,q) \propto e^{\tau(q)}$ is called a mass exponent function, which is the characteristic function of fractal behaviour. If $\tau(q)$ is a straight line, the object about q of study is the single fractal. If $\tau(q)$ is a convex function of q, the research object has multifractal features.

Define:

$$D(q) = \begin{cases} \dfrac{\tau(q)}{q-1} & q \neq 1 \\ \lim\limits_{\lambda \to 0} \dfrac{\sum_i e(\lambda,i) \cdot \log e(\lambda,i)}{\log \lambda} & q = 1 \end{cases} \quad \text{... (15)}$$

where the generalized fractal dimension $D(q)$ has different meanings with different values q.

There are two kinds of representation methods of $D(q)$~q and $f(\alpha)$~ α. The method of computing the singular spectrum to calculate the multifractal spectrum is called direct calculation method. Multifractal can also be calculated by the generalized dimension. There are three methods to calculate the generalized fractal dimension: box counting method, fixed radius method and fixed mass method. The box counting method is used in this paper.

When $\tau(q)$ and $f(\alpha)$ are differentiable, the generalized fractal dimension $D(q)$ and multi-fractal spectrum $f(\alpha)$ meet the Legendre transform:

$$\begin{cases} \alpha(q) = \dfrac{d\tau(q)}{dq} \\ f(\alpha) = q\alpha(q) - \tau(q) \end{cases} \quad \text{... (16)}$$

According to the Legendre transform, we can know that if the object of the study is a single fractal, the function $f(\alpha)$ is a certain value, and if it is the multi-fractal, the function $f(\alpha)$ is generally a single peak curve, and the interval meet that $f(\alpha)\geq 0$ is the range of $\alpha \in [\alpha_{min}, \alpha_{max}]$. It is satisfied the following relationship:

$$f(\alpha_{\min}) = f(\alpha_{\max}) = 0$$

There must exists an a_0 between $\alpha \in [\alpha_{min}, \alpha_{max}]$, which can establish the relationship that meets $f_{max} = f(\alpha_0)$. Then the multi-fractal degree of shale pore structure is expressed as follows.

Multifractal maximum value is :

$$f_{max} = f(\alpha_0) \quad \alpha_0 \in [\alpha_{min}, \alpha_{max}] \quad \text{... (17)}$$

Multifractal spectrum width is:

$$W = \alpha_{\max} - \alpha_{\min} \quad \text{... (18)}$$

Nearby $\alpha = \alpha_0$ where is f_{max}, a quadratic function of the fitting with minimal quadratic fitting method is:

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C \quad \text{... (19)}$$

where α_L is the half width of shale pore fissure structure singular spectrum $f(\alpha)$ and α_R is the right width, so $B = \alpha_L - \alpha_R$ denotes the asymmetry degree of the shale fracture in the multifractal spectrum curve. When $B = 0$, the multifractal spectrum curve of crack is symmetric; when $B>0$, the peak of crack multifractal spectrum curve is to the left; when $B<0$, the peak of crack multifractal spectrum curve is to the right. In general, when f_{max} is larger, the width of the multifractal spectrum curve W is larger and the multifractal spectrum curve $f(\alpha)$ symmetry is better, the multifractal of shale rock cracks is stronger. Therefore, the physical significance of $f(\alpha)$ in this paper is a measure of the degree of complexity, irregular and uneven of the shale fracture fractal structure.

3.2. Brittleness characterization methods based on multifractal

The higher the Young's modulus is, the lower is the Poisson's ratio. The dimensionless elastic modulus and Poisson's ratio are obtained by setting the maximum, minimum elastic modulus and Poisson's ratio of rock, and the brittleness of rock is obtained through the average value method. Specific calculation methods are as follows [15]:

$$E\_BRIT = \left(\frac{E - E_{\min}}{E_{\max} - E_{\min}} \right) \times 100 \quad \text{... (20)}$$

$$\nu\_BRIT = \left(\frac{\nu - \nu_{\max}}{\nu_{\min} - \nu_{\max}} \right) \times 100 \quad \text{... (21)}$$

$$BRITavg = \frac{1}{2}(E\_BRIT + \nu\_BRIT) \quad \text{... (22)}$$

where v is Poisson's ratio of some point; E is Young's elastic modulus of some point; E_{min} is the minimum value of Young's modulus, general is 1.0 GPa; E_{max} is the maximum value of Young's modulus, general is 8.0 GPa; v_{min} is the minimum value of Poisson's ratio, general tale 0.15; v_{max} the maximum value of Poisson's ratio, general tale 0.4.

Considering the shale fracture distribution caused material in homogeneity and anisotropy [16]. The relative fractal dimension δ_D is defined as:

$$\delta_D = \frac{f(\alpha_0)}{3} = \frac{f_{\max}}{3} \quad \text{... (23)}$$

Considering the influence of micro cracks on the mechanical parameters of rock mass fracture, according to the Lemaitre strain equivalence hypothesis and the damage constitutive relation of rock mass, there is:

$$D = \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}} \quad \text{... (24)}$$

At the same time, formula (20), and formula (23)~(25) can be obtained, the rock brittleness of the rock mass can be expressed as:

$$E\_BRIT = \frac{E - E_{\min}\left(1 - \frac{f_{\max}}{3} \cdot \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}}\right)}{\left(1 - \frac{f_{\max}}{3} \cdot \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}}\right)(E_{\max} - E_{\min})} \times 100 \quad \text{... (25)}$$

Formula (25) is a method to characterize the brittle rock mass with multiple fractal characteristics.

4. The characterization method of shale brittleness based on multiple linear regression

Multiple linear regression analysis means that when a phenomenon is affected by many factors, it analyzes the relationship between the explanatory variables and its influencing factors, and establishes the multiple linear regression model of the explanatory variables.

According to the parameters of the model, analyzing the estimated parameters and regression equation, and the regression model is used to predict and analyze. The multivariate linear regression model contains multiple explanatory variables, and they play an important role in the interpretation of Y together. To investigate the effect of one of the explanatory variables on Y, it is necessary to assume that the other variables remain unchanged. Therefore, the regression coefficients of the multiple linear regression model is partial regression coefficient, which reflects when other variables in the model remains the same, one of the explanatory variables effects the dependent variable Y [17]. The general form of multiple linear regression model is:

$$Y = \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \text{LL} \quad + \beta_{jj} Z \quad + \varepsilon \quad \text{... (26)}$$

where β_j (j = 1,2,L,k) is regression coefficient; Y is an explanatory variable; Z_j (j = 1, 2, L, k) is a significant explanatory variable of K ($k \geq 2$); e is a random item which

TABLE 1: MINERAL COMPOSITION TABLE OF THE BLOCK

Core	Mineral content %							
	Clay	Quartz	Feldspar	Analcite	Calcite	Dolomite	Siderite	
Rock sample1#	22.9	23.1	9.3	3.1	8.6	29.8	0.6	2.6

TABLE 2: FRACTURE DEVELOPMENT CHARACTERISTICS OF THE BLOCK

Well name	Number of coring	Length m	Fracture section	Fracture number	Cracks through length m	Cracks through rate %	Linear fracture density N/m	Fracture aperture mm	Fracture dip angle
Well 1#	2	9.91	16	25	1.39	14.0	2.5	0.1-2.0	60º-80º

reflects the influence of various errors.

Assuming multivariate regression function is:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 z_{1i} + \hat{\beta}_2 z_{2i} + \cdots + \hat{\beta}_k z_{ki} \quad \text{... (27)}$$

Regression residual is:

$$\varepsilon_i = Y_i - \hat{Y}_i \quad \text{... (28)}$$

Due to the observation value of the N period, the model actually contains n equations, written in matrix form:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & z_{11} & z_{21} & \cdots & z_{k1} \\ 1 & z_{12} & z_{22} & \cdots & z_{k2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & z_{1n} & z_{2n} & \cdots & z_{kn} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \quad \text{... (29)}$$

where $[Y_1 \quad Y_2 \quad \cdots \quad Y_n]^T$ is observed vector of the explanatory variables; $[\beta_0 \quad \beta_1 \quad \beta_2 \quad \cdots \quad \beta_k]^T$ is population regression parameter vector; $[\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_n]^T$ is random error term vector.

Because the establishment or choice of multivariate linear model is relatively subjective, and there are some errors in the theory and experience. In order to guarantee the variable relationship analysis accords with the basic provisions of the multivariate linear regression analysis and the validity of the regression analysis, the sample determination coefficient is used as the index to test the regression equation and the fitting degree of sample values in this paper. R^2 ($0<R^2<1$) is bigger, the independent variable to the dependent variable explanation degree is higher, the regression equation and the sample fitting is better; On the contrary, the explanatory variables of the independent variables are relatively low, and the regression equation is not fitting to the sample. The formula is:

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \left(Y_i - \hat{Y}_i\right)^2}{\sum \left(Y_i - \bar{Y}_i\right)^2} \quad \text{... (30)}$$

where ESS is the regression square sum of fitting results; RSS is the sum of squared residuals of the fitting result, which reflects the effect of random error; TSS is the sum of squared deviations of fitting results; $\bar{Y}_i$ is sample average, that is $\bar{Y}_i = \frac{\sum Y_i}{i}$

5. Example analysis

Coring data come from block A interval B of Songliao basin, the mineral composition can be observed as shown in Table 1.

After CT scanning of block A, it can obtain fracture distribution when 30s, 90s, 120s and 180s respectively, as shown in Fig. 4.

By CT scanning, the paper used MATLAB to reconstruct three-dimensional core, and obtained 3D spatial distribution of Sample 1# as shown in Fig.5.

Rock sample 1# in different length of multifractal parameters are as shown in Table 3. Relationship between statistical moment order and generalized fractal dimension is shown in Fig. 6, multifractral spectrum of rock mass fracture distribution characteristics is shown in Fig.7.

The statistical results show that the fracture size of the shale is dominated by the micro cracks of 0-0.8cm, and the fractures are distributed in multiple scales. The spatial distribution of cracks is divergent, and the connectivity is poor. The spectral width range of 0.6-0.8cm in the sample is the largest, which indicates that the 0.6-0.8cm size of the crack is the most widely distributed in the rock mass. The multifractal spectrum of 0.6-0.8cm is the largest, which shows that the 0.6-0.8cm size of the fractures in the reservoir is more complex and uneven distribution. The singular spectrum width and maximum value of the pore structure of the rock sample are larger, and the fractal spectral function is

TABLE 3: MULTIFRACTAL PARAMETERS OF DIFFERENT FRACTURE LENGTH

Rock sample	Sample1 #			
	0-0.2 cm	0.2-0.4 cm	0.4-0.6 cm	0.6-0.8 cm
a_{max}	2.1125	2.1432	2.3124	2.3715
a_{min}	0.2543	0.2649	0.2134	0.1542
W	1.8582	1.8783	2.0990	2.2173
f_{max}	1.4321	1.5276	1.5631	1.6364

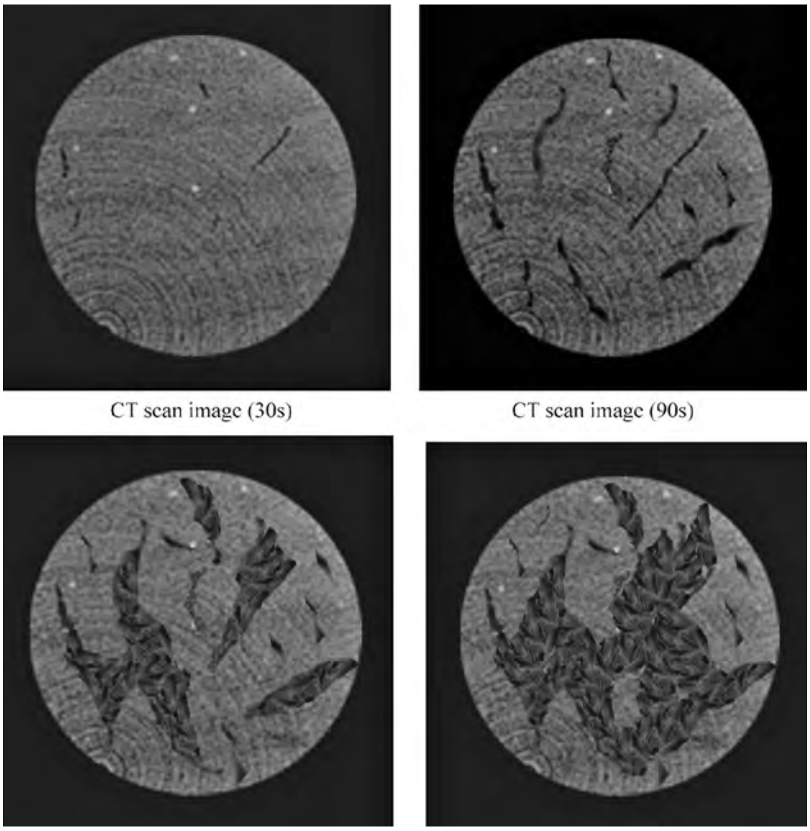

Fig.4 Fracture distribution pictures after CT scanning

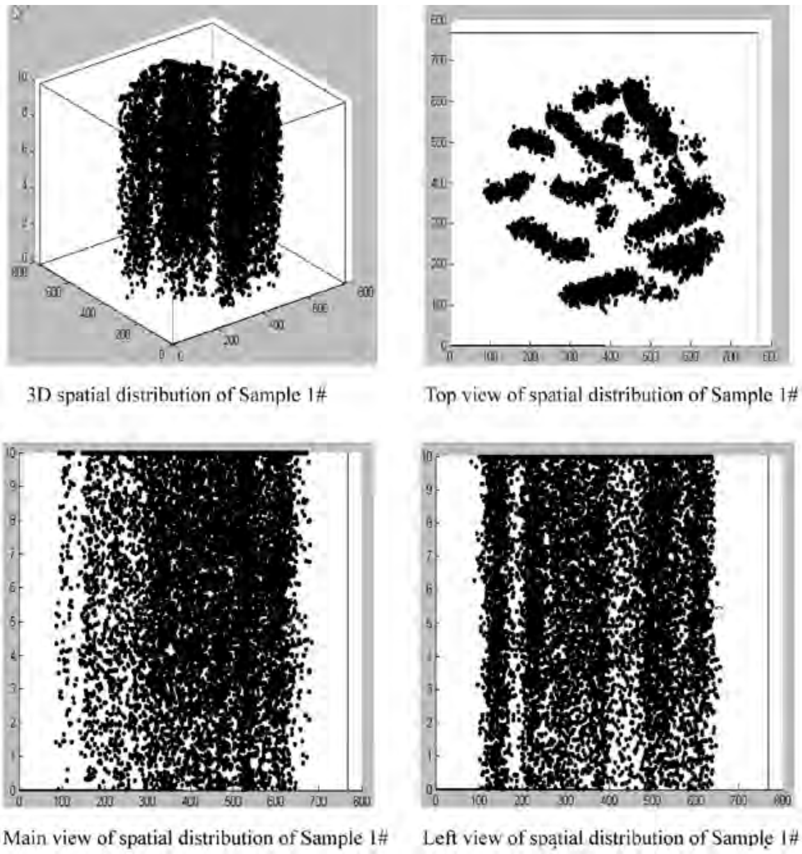

Fig.5 Pores and fractures 3D distribution characteristics in shale rock mass

asymmetric, so the shale in this area has obvious multifractal characteristics.

The elastic modulus and Poisson's ratio of core are measured by uniaxial compression test, and the test data are shown in Table 4.

Base on multiple linear regression method, shale mineral composition and mechanical parameters are shown in Table 5.

The relationship of elasticity modulus, Poisson's ratio and mineral composition are shown as formula (31) ~ (32).

$$E=15.576+0.148x_{quartz}+0.080x_{potassium\ feldspar}+0.145x_{anorthose}$$
$$\mathrm{V}+0.129x_{calcite}+0.262x_{dolomite}+0.200x_{analcite}-0.171x_{clay}$$
... (31)

$$\nu = 0.254715 - 0.000204x_{quartz} + 0.001197x_{potassium\ feldsper} - 0.000108x_{anorthose} + 0.001218x_{calcite} + 0.000271x_{dolomite} - 0.000343x_{siderite} - 0.002222x\ pyrite + 0.001069x_{clay}$$
...(32)

In order to verify the accuracy and reliability of the calculation results, the analysis of the standardized residuals are shown in Figs.8 and 9.

According to the relationship among elastic modulus, Poisson's ratio and the elements of the composition of rocks and minerals, considering the regularity of multifractal distribution in rock mass in different scale fractures in shale rock, mineral composition and rock mechanics parameters of rock mass, establish shale brittleness index characterization method based on multiple linear regression, as shown in formula (33).

$$hg\_BRIT=\left[\frac{\dfrac{\begin{pmatrix}15.576+0.148x_{quartz}+0.080x_{potassium\,feldspar}+0.145x_{anorthose}\\+0.129x_{calcite}+0.262x_{dolomite}+0.200x_{analcite}-0.171x_{clay}\end{pmatrix}}{\left(1-\dfrac{f_{max}}{3}\dfrac{\alpha_0-\alpha_{min}}{\alpha_{max\,min}-\alpha}\right)}-E_{min}}{E_{max}-E_{min}}+\frac{\begin{pmatrix}0.254715-0.000204x_{quartz}+0.001197x_{potassium\,feldspar}-0.000108x_{anorthose}+0.001218x_{calcite}\\+0.000271x_{dolomite}-0.000343x_{siderite}-0.002222x_{pyrite}+0.001069x_{clay}-\nu_{max}\end{pmatrix}}{\nu_{min\,max}-\nu}\right]\times 100$$
... (33)

where hg_BRIT is brittleness index based on multiple linear regression.

6. Conclusions

(1) Through the multifractal theory, the multiple distribution law of shale reservoir fracture is calculated. The results show that the fracture

TABLE 4: CORE MINERRAL COMPOSITION AND THE MECHANICAL PARAMETERS DETERMINATION RESULTS

	Mineral type and content (%)								Clay (%)	Elasticity modulus	Poisson's ratio
	Quartz	Potassium feldspar	Anorthose	Calcite	Dolomite	Analcite	Siderite	Pyrite			
1	16.30	3.10	26.80	7.30	20.20	3.20	2.10	2.50	18.50	30.23	0.293
2	19.70	3.60	7.30	8.90	9.50	29.10	1.80	1.90	18.20	32.79	0.283
3	14.80	3.80	16.60	4.40	21.70	12.10	1.40	2.80	22.40	24.65	0.300
4	33.90	3.20	13.80	0.00	19.80	8.90	0.00	0.00	20.40	26.98	0.286
5	21.10	4.00	15.90	5.20	15.10	13.60	1.50	2.00	21.60	26.74	0.279
6	19.50	3.70	12.50	1.10	25.40	14.00	0.00	0.00	23.80	24.88	0.279
7	17.70	5.40	27.30	0.00	15.30	3.30	0.00	1.50	29.50	22.56	0.276
8	9.60	4.30	18.50	0.00	8.80	38.10	3.50	0.00	17.20	22.56	0.281
9	10.60	7.00	11.70	2.00	26.70	16.00	3.00	1.80	21.20	26.27	0.267
10	14.00	5.30	24.20	11.00	18.00	5.00	1.30	2.40	18.80	20.93	0.281
11	25.30	4.10	6.90	2.40	5.30	28.20	5.60	0.00	22.20	22.10	0.286
12	1.80	2.70	16.40	1.10	41.30	10.90	15.10	0.00	10.70	29.53	0.271
13	12.10	7.30	33.40	0.00	18.90	4.00	8.70	2.00	13.60	31.63	0.262
14	19.60	6.50	9.60	2.00	13.90	30.90	0.00	0.00	17.50	30.00	0.271
15	12.40	5.80	6.30	1.70	5.40	42.90	1.90	0.00	23.60	28.84	0.286
16	4.10	14.80	36.00	3.90	24.10	0.00	0.00	4.20	12.90	29.53	0.281
17	6.80	11.40	49.30	0.00	13.50	0.00	2.20	1.50	15.30	26.28	0.281
18	8.20	6.70	41.50	1.90	17.70	4.80	3.10	0.00	16.10	28.60	0.273
19	9.50	10.10	11.30	3.60	35.90	4.50	1.60	2.70	20.80	23.49	0.310
20	37.50	2.00	19.20	14.40	4.10	4.50	1.30	4.60	12.40	23.95	0.262
21	19.80	5.90	10.30	2.00	10.80	29.80	4.80	0.00	16.60	19.77	0.286
22	6.10	8.40	46.80	2.40	13.70	1.70	4.90	0.00	16.00	19.53	0.290

TABLE 5: SHALE MINERAL COMPOSITION AND MECHANICAL PARAMETERS OF MULTIPLE LINEAR REGRESSIONS ANALYSIS

Influence factor	Coefficient	Quartz	Potassium feldspar	Anorthose	Calcite	Dolomite	Analcite	Siderite	Pyrite	Clay
Elasticity modulus	15.576	0.148	0.080	0.145	0.129	0.262	0.200	0	0	-0.171
Poisson's ratio	0.254715	-0.000204	0.001197	-0.000108	0.001218	0.000271	0	-0.000343	-0.002222	0.001069

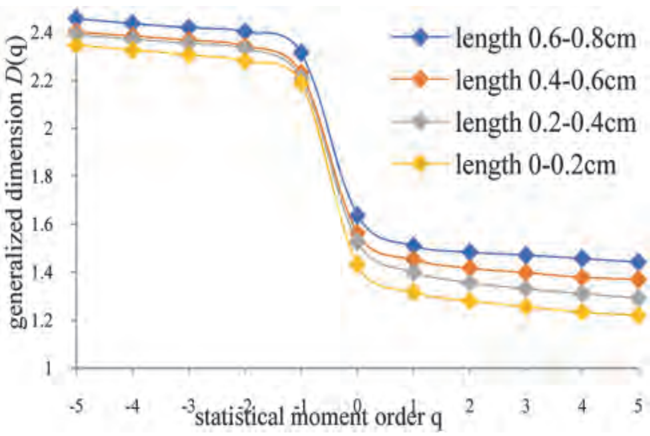

Fig.6 Relationship between statistical moment order and generalized fractal dimension (sample 1#)

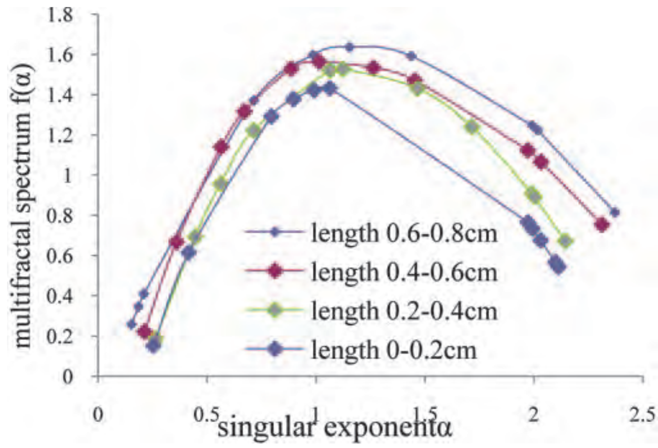

Fig.7 Multifractal spectrum of rock mass fracture distribution characteristics (sample 1#)

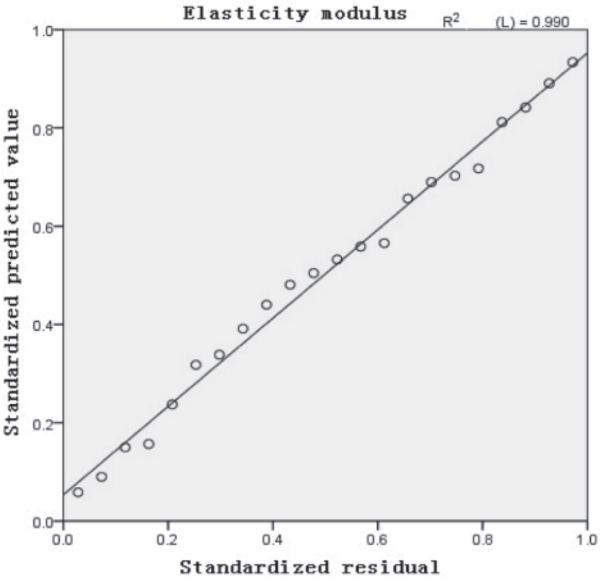

Fig.8 Probability distribution of elastic modulus

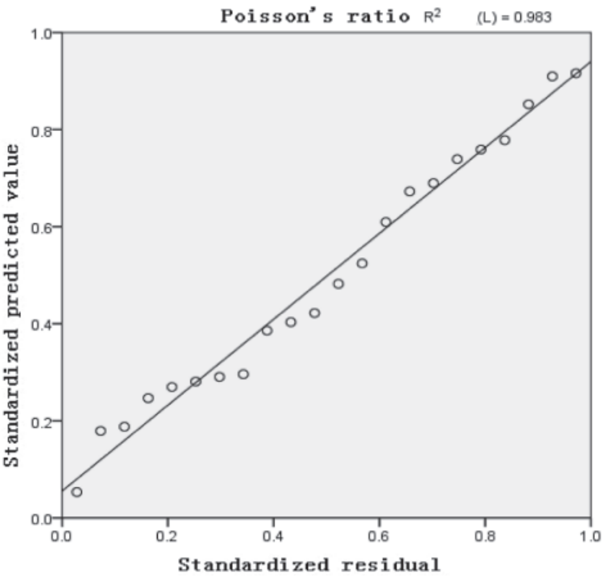

Fig.9 Probability distribution of Poisson's ratio

distribution of shale reservoir with different length has multifractal law.

(2) Based on the analysis of the brittle characteristics of shale, considering the multifractal distribution characteristics of fractures in different scales, the method of calculating the brittleness index of shale rock mass is established based on the multiple linear regression method.

(3) The results show that the new method based on multiple regression method to calculate rock brittleness is foundation for the subsequent research on unconventional oil pressure cracking.

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