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Finite element analysis of high performance steel reinforced concrete T-beam bridge

Highway traffic is the lifeline of the national economy, and the bridge is the throat of the road, so the well used status of bridge is an important determinant factor of highway traffic safety. With the development of metallurgical and welding technology, high performance steels have been widely used in large span concrete bridge structures. Based on the theory of finite element method and ANSYS software, Lianhuo highway prestressed concrete continuous beam bridge is taken as the research object, and a three-dimensional finite element model of the bridge is established to analyse the stress characteristics, load effect, internal force of the structure in the ultimate limit state of bearing capacity and serviceability under different conditions. The results can provide a reliable basis for the design of the bridge.

Key words: *High performance, T-beam bridge, ultimate limit state, serviceability limit state, modal analysis.*

1. Introduction

Bridge is one of the important traffic implements in our country and plays an important role in people's production and life. The development of bridge construction is closely related to economic development. With the continuous development of China’s modernization, it is necessary to build a large number of bridges to meet the needs of transportation. China’s bridge construction projects continue to increase to meet people's transport needs in large extent. However, the improvement of bridge technology mainly depends on the design theory, mechanical analysis methods, and the level of building materials industry and construction technology development. T-beam bridge is built most in our country, this kind of bridge has played an important role in improving the traffic. Prestressed T-type concrete bridge structure is simple, clear force, saving materials, erection convenient, greater ability to cross and other advantages. In bridge construction, compared to the traditional technology, construction technology of precast prestressed T type bridge can not only accelerate the construction speed, but also makes the main beam deadweight more light, it has been widely popularized and applied in many bridge construction. In the specific application process, the method of precast construction is different, and the quality of construction is not same. Therefore, in order to improve the overall quality of the bridge construction, we have to do some transformation on the prefabricated construction of prestressed T-beam bridge [1-2].

For more than forty years, a large number of prestressed concrete bridges have been built on the highway, especially in the large span bridges. Such as China has built Shanghai Yangpu Bridge (span 602 meters) and other seven cable-stayed bridge, which span more than 400 meters. It is represented that China's cable-stayed bridge technology has entered the world’s leading level; continuous steel frame bridge after the Huangshi bridge main span is 250 meters, Humen Bridge is 270 meters, it is the largest span in the world. Panzhihua Jinsha river bridge, Qiantang river bridge and other railway bridges show that China's prestressed concrete technology of railway bridge has reached the advanced level in the world [3].

Due to the existence of prestressing force, the static and dynamic characteristics of the structure will change. For the effect of prestressing effect on the static of the structure, it already has a more in-depth study and a more reasonable conclusion; For example, the pre-stress not only can make the tensile and compressive strength of the material have the full use and save the materials, but also to improve the structure of the bearing capacity, increase structural stability, reduce the deflection of the structure. The influence of the prestress on the dynamic characteristics is worth to study in-depth. Prestressed beam as a bending component, the free vibration frequency reflects the vibration characteristics of the bridge, the design of wind resistance, seismic and damping, and the structural dynamic calculation of bridge must be combined with the response of structure dynamic. However, the most fundamental and crucial point is analyzed the natural frequencies and modes of the bridge structures. The prestressing effect of the bridge will change the stiffness, section characteristics and mass characteristics, thus lead to the change of the structural vibration characteristics [4-5].

This study mainly combined with the stress

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characteristics, load effect, stress and internal force of the structure, and other computing content of the prestressed concrete continuous bridge structure design, to simulation of the limit state of bearing capacity, serviceability limit state and the mode frequency change under different prestressed of the bridge. Study of resonance phenomena in practice can solve many practical problems. According to the predictable dynamic performance of the product, it is possible to estimate the vibration, noise intensity and other dynamic problems before the product is manufactured, and eliminate or suppress these problems by changing the shape of the structure.

2. The basic theory

2.1. Four strength theory

(1) Maximum tensile stress theory (first strength theory): The theory think that the cause of brittle fracture failure of the material is the maximum tensile stress. Whatever the stress state, as long as the maximum tensile stress in the component at one point reaches the ultimate stress under the uniaxial stress state, the material will undergo brittle fracture. So, the dangerous point of the components in the complex stress state appear brittle fracture failure conditions is:

$$\sigma_1 = \sigma_b \quad \sigma_b / s = [\sigma] \qquad \text{...} \quad (1)$$

So, the condition of strength established by first strength theory is:

$$\sigma_1 \le [\sigma] \qquad \text{...} \quad (2)$$

(2) Theory of maximum stretching strain (second strength theory): This theory holds that the maximum elongation strain is the main cause to fracture, no matter what stress state, as long as the maximum elongation strain reaches the limit value under the uniaxial stress state, the material will appear brittle fracture failure. From the generalized Hooke law:

$$\varepsilon_1 = [\sigma_1 + \mu(\sigma_2 + \sigma_3)] / E \qquad \text{...} \quad (3)$$

So, the strength condition is:

$$\sigma_b = \sigma_1 - \mu(\sigma_2 + \sigma_3) \qquad \text{...} \quad (4)$$

(3) Maximum shear stress theory: This theory holds that the maximum shear stress is the main cause to yield. No matter what the stress state, as long as the maximum shear stress reached uniaxial stress state of the ultimate shear stress, the material will yield. According to the axial tensile formula on the oblique section of the stress can be seen:

$$\tau_0 = \sigma_s / 2 \,(\sigma_s - \text{the normal stress in the cross section}) \qquad \text{...} \quad (5)$$

From the formula:

$$\tau_{max} = (\sigma_1 + \sigma_3)/2 \qquad \text{...} \quad (6)$$

According to the third strength theory:

$$(\sigma_1 + \sigma_3) \le [\sigma] \qquad \text{...} \quad (7)$$

(4) Energy of form-changed criterion: The theory suggests that the shape change ratio is the main factor causes the yield failure of the material. No matter what the stress state, as long as the component at a point in the shape change achieve unidirectional stress state limit, the material will yield damage. The conditions to plastic damage, as the fourth strength theory conditions:

$$\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3\right)^{2} \le [\sigma] \qquad \text{...} \quad (8)$$

In the formula, s_1 is the maximum tensile stress, s_b is the ultimate stress under uniaxial stress, $[s]$ is the allowable stress. e_b is maximum elongation line strain. t_{max} is the maximum shear stress. t_0 is the ultimate shear stress.

2.2. Finite element theory

The advantage of using the finite element method for dynamic analysis [6] is that the material characteristic constants of the individual elements can be varied arbitrarily. After obtaining the stiffness matrix, the mass matrix and the damping matrix for each element, as long as a simple superposition, then can get the overall stiffness matrix, damping matrix and mass matrix. Another important advantage is that the stiffness matrix, mass matrix and damping matrix are highly sparse, and it is easy to be arranged in a strip matrix, so that it is easy to calculate.

The core of the finite element method is to establish the element stiffness matrix [7], with the stiffness matrix, and properly combined, the balance equation can be obtained. In the calculation of plane problem of elasticity, the stiffness matrix of element is established by intuitionist method. The advantage is that it is easy to understand and easy to construct clear mechanical concept. However, this method also has drawbacks: on the one hand, it is difficult to establish the element stiffness matrix for more complicated elements; on the other hand, it cannot give proof of convergence. Using energy principles into finite element method, you can overcome these shortcomings. The energy principle provides a powerful tool for establishing the basic formula of the finite element method and gives a strong proof. In all kinds of energy principle, the principle of virtual displacement is the most convenient.

Finite element method [8] is mainly use the finite element to discretize the continuum, doing piecewise interpolation to the finite element to solve a variety of mechanical and physical problems. The finite element method discretizes the continuum into finite units: The unit of the skeletal structures is each rod; the unit of the continuum is each shape (e.g., triangular, tetrahedral, and hexahedral). The field function of each element is a simple field function containing only the parameters of the finite undetermined nodes. The set of these field functions can approximate the field functions of the whole continuum. The finite element method is used to solve the algebraic equations, and the numerical solution of the finite element method can be obtained by solving the finite element method. Finite element method has been used to solve linear and non-linear problems, and establish a variety of finite element models, such as coordination, uncoordinated, mixed, hybrid, and coordinated element and so on.

2.3. Prestressed modal analysis theory

The stress in the structure may cause the stiffness of the structure to change, based on the modal analysis of the conventional ANSYS software, the mode of the structure under the influence of the different prestressing force of the concrete T-shaped beam is analyzed by using the prestressed modal analysis. The prestressed modal analysis is used to analyze the natural frequency of the prestressed structure, modal analysis method ANSYS has six ways: subspace, block lanczos, power dynamics, reduced, un-symmetric and damped.

3. Engineering survey

The research object is prestressed concrete continuous beam bridge of Lianhuo expressway, the bridge is located on the plains of the highway, across the river. According to the highway design requirements, the upper part is three-span continuous beam bridge, span is 30 m+30 m+30 m. Design load is the road-I level, the concrete density is 2500kg/m^3, the elastic modulus is 3E10 Pa, Poisson's ratio is 0.167; the reinforcement density is 7800k g/m^3, the elastic modulus is 2E11 Pa, and the Poisson's ratio is 0.3.

4. Finite element model

The SOLID65 element is used to simulate concrete solid, which can obtain the geometric model of plain concrete with little error. The LINK8 element is used to simulate the steel bar, and the MPC184 model is used to simulate the bearing unit, then the final geometry model is built.

The SOLID 65 unit has the performance of crack and crush, for the 3D model containing or not containing steel reinforced concrete, it is not only can simulate crack (three orthogonal directions), crushing, plastic deformation and creep of concrete, but also can simulate tensile, compression, plastic deformation and creep of bar. Each unit has 8 nodes, each node has 3 degrees of freedom. The prestressed reinforcement is simulated by the element link 8, and the unit only can bear the axial tension and compression. Each unit has 2 nodes, each node has three degrees of freedom, which can simulate the truss, cable, steel spring structure [9-10]. MPC184 unit includes a class of commonly used multi point constraint units using Lagrange multiplier method to implement motion constraints. These units can be simply divided into “constraint units” or “connection units”. Meshing the model after defining the material properties, it totally have 99015 nodes. The X axis is the bridge transverse direction, the Y axis is the bridge vertical direction, and the Z axis is the bridge longitudinal direction. The finite element model and longitudinal prestressing tendons are shown in Figs.1 and 2.

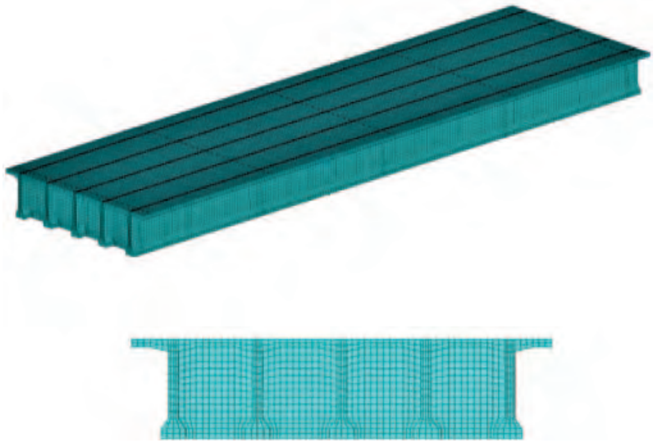

Fig.1 Finite element model of bridge

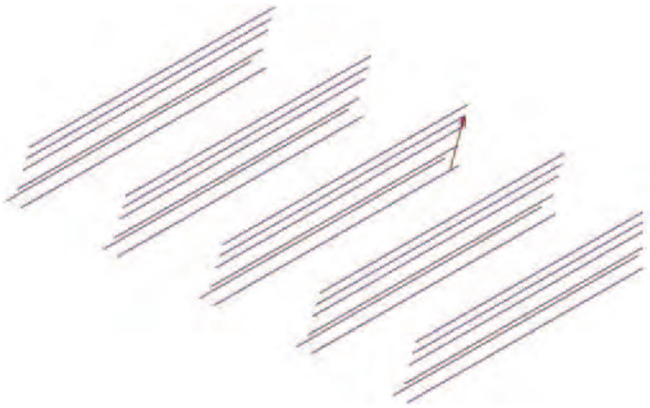

Fig.2 Longitudinal prestressing tendon

5. Three-dimensional finite element static analysis of T-type bridge

During the operation of the bridge, the following two conditions should be considered (1) Deadweight + double lane partial load; 2) deadweight + double lane full load. The displacement and stress of T type bridge structure under different working conditions are obtained, as shown in Tables 1 and 2. Because condition 1 is asymmetric load, the force in the operation is more complex, and it is a typical working condition. Figs.3~4 show the deformation and the distribution of the major principal stress of the bridge under the loads of condition 1 during the operation.

From Table 1, the maximum deformation in the Y-direction of condition 1and 2 are 25.313mm and 27.655mm, and they all appear in the mid-span. Y direction deformation larger than the Z direction deformation are 452%, 336%, and 517%, 11520% higher than that in X direction, obviously X, Z direction of deformation is negligible. The deformation at the end of the bridge is small or zero, due to the bearing restraint. It can be seen from Figure 3 that under the road eccentric load, the mid-span deflection is the largest, and the closer the deflection, the smaller the deflection. The maximum

Table 1: The maximum deformation of the bridge at different angles

Condition	Bridge displacement (mm)	X-direction (mm)	Y-direction (mm)	Z-direction (mm)
Unbalance load	25.406	4.105	25.313	4.584
Full load	27.833	0.238	27.655	6.342

deformation of the bridge is 25.406mm, the deformation is small, due to the anti-arch effect caused by prestressing and can offset part of the deformation, so that the normal work of the bridge does not produce too large deflection, the bridge has more reliable performance. According to the specification, the maximum vertical deflection in the main spans should not exceed L/600 [11], the deflection of the bridge is 0.05m, the maximum allowable deflection is greater than the bridge deflection, so from the perspective of adaptability that the bridge is in normal use state.

From Table 2, the T-beam strength is verified according to the first strength theory, the corresponding stress of condition 1 and condition 2 first strength theory is 35MPa and 28.2 MPa respectively, significantly greater than the Z axis direction stress 14.1 MPa, 15.1 MPa, the strength is meet the requirements.

The T-beam strength is verified according to the second strength theory, the corresponding stress of the second strength theory is 14.1 MPa and 27.8 MPa, respectively, and the strength satisfies the requirement.

The equivalent stress of the first strength theory is larger than second strength theory, it can be seen that the second strength theory considers three principal stresses, which are safer than the first principal stress. Stress concentrate in the cross-diaphragm and T-beam interface, bearing, etc., where significant changes in the shape, the stress will have a significant in a small area. This is a stress concentration phenomenon.

6. Analysis of dynamic characteristics of T-type bridge

A. Table 3 shows the modal frequencies of the first 10 orders of gravity and different prestressing conditions, with prestresses of 50 KN, 100 KN, 150 KN, 193.9 KN.

Choose the modal of continuous T-type bridge under the condition of maximum prestress. From the finite element analysis theory, the structure of the natural frequency is more, but the first 10 order is more important, so extract the frequency of the first 10 orders and the corresponding vibration mode to analysis, the frequency results are shown in Table 3.

As can be seen from Table 3: Analyze the modal for the same model, there is no change in modal frequency, period and amplitude under different conditions of prestressing. This is because the effect of the modal is the mass and stiffness distribution, and the structural characteristic details impact on the modal is slight. As is known that prestressing can improve the stiffness of the steel bar, while the influence of the prestressing on the mode is neglected, so the ability of prestress to improve the stiffness of concrete members is still very weak [12].

In general, the vibration response stimulated by external condition is a combination of the various order formation, while the mode is the inherent vibration characteristics, each mode has a specific natural frequency, damping ratio and modal formation. In the

Table 2: Stress summary of the bridge

Condition	Major principal stress (MPa)	Second principal stress (MPa)	Third principal stress (MPa)	Z direction stress (MPa)
Unbalance load	35	14.1	4.91	32.6
Full load	28.2	15.1	5.51	26.2

Table 3. The first 10 orders of frequencies under different prestressing conditions

Orders	50KN			100KN			150KN			193.9KN		
	Frequency	Cycle	Amplitude	Frequency	Cycle	Amplitude	Frequency	Cycle	Amplitude	Frequency	Cycle	Amplitude
1	4.185	0.234	2.486	4.1852	0.239	2.486	4.1852	0.239	2.486	4.185	0.239	9.992
2	4.7637	0.21	3.884	4.7636	0.210	3.884	4.7636	0.210	3.884	4.764	0.21	3.884
3	11.079	0.09	3.249	11.079	0.09	3.249	11.079	0.09	3.249	11.08	0.09	3.249
4	12.889	0.08	2.261	12.889	0.08	2.261	12.889	0.08	2.261	12.89	0.078	2.261
5	15.141	0.07	3.8	15.141	0.07	3.8	15.141	0.07	3.8	15.14	0.07	3.8
6	18.986	0.05	2.42	18.986	0.05	2.42	18.985	0.05	2.42	18.99	0.05	2.42
7	21.153	0.047	4.078	21.152	0.047	4.078	21.152	0.047	4.078	21.15	0.047	0.78
8	26.556	0.038	8.096	26.556	0.038	8.096	26.555	0.038	8.096	26.56	0.038	8.063
9	28.249	0.035	8.476	28.249	0.035	8.476	28.248	0.035	8.476	28.25	0.035	8.476
10	28.5	0.035	8.578	28.499	0.035	8.578	28.499	0.035	8.578	28.50	0.035	8.579

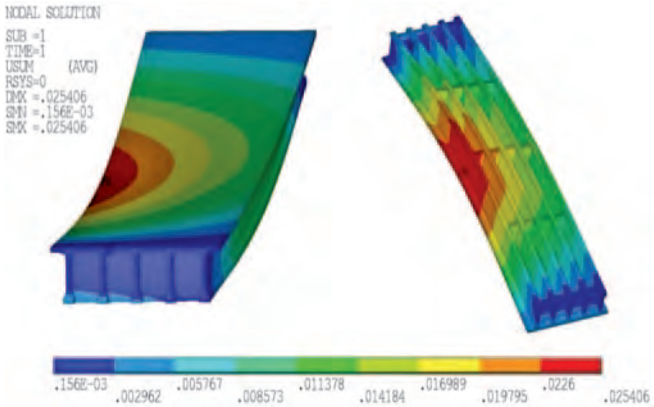

Fig.3 Total deformation of the T-bridge

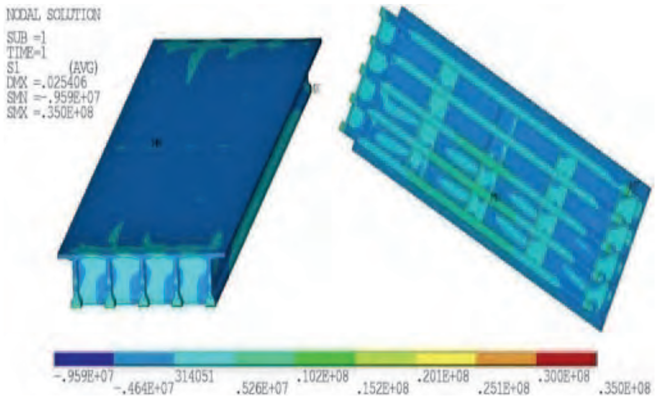

Fig.4 Major principal stress

dynamic knowledge the self-vibration frequency of the structure is related to its stiffness K and quality M[13]. If the structure weight unchanged, the greater the frequency, the greater the structural stiffness. Under the influence of gravity and prestressing force (193.9KN), the higher the modal order, the higher the frequency, the smaller the period, but not the linear decrease. The order and the formation are corresponding, how many orders there are, how many order modes. In theory, the first-order formation is the same as the natural frequency, the amplitude is the largest, the frequency is smallest, which is resonance. It can be seen that the amplitude decreases first and then increases with the increase of the order, while the amplitude should be reduced in theory. This is because only finite number of natural frequencies (the same as the number of unrestricted degrees of freedom) can be obtained in finite element calculation according to the experience, and the higher the order, the greater the error. The first few order modes are significant to the practical structure. So, the results are accord with the actual requirements.

B. T-type bridges in the first 10 order modes of gravity and pre stress (193.9KN)

The first 6 order modes of the bridge structure are obtained by calculating, as shown in Figs.5 to 10.

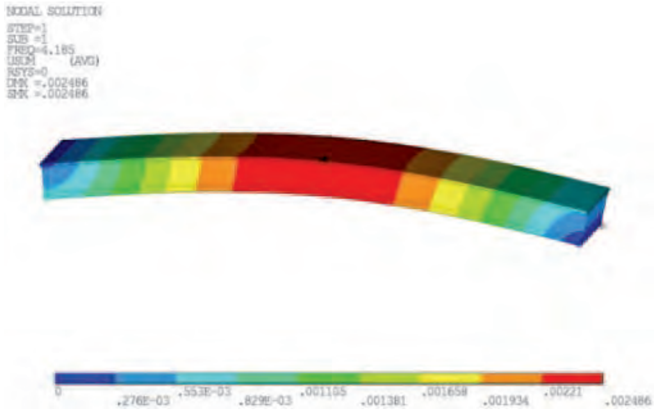

Fig.5 The first order vibration mode

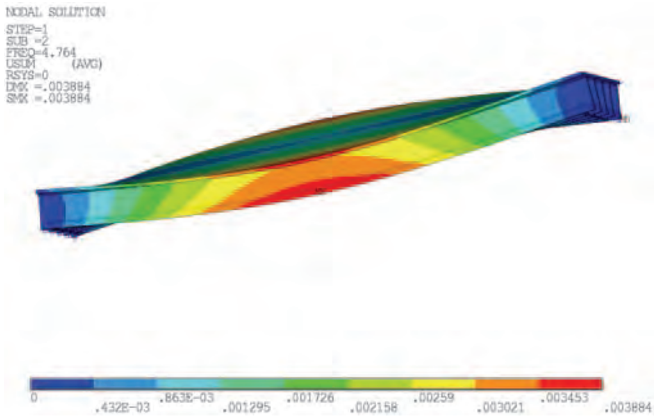

Fig.6 The second order vibration mode

By analyzing the first order mode, the frequency of the first mode is 4.185 Hz. This is the mode of gravity and prestressing (193.9 KN). Its vibration mode is transverse vibration, lateral stiffness is weak, its frequency has some changes, the results show that the prestress has a certain influence on the natural frequency of the bridge.

The second-order vibration begins to reverse, the bridge body have bending deformation, bending in the middle is more lager, the relative displacement is the largest, it is become smaller at two sides, because the bearing is rigid constraint, the displacement is zero. Rod deformation is

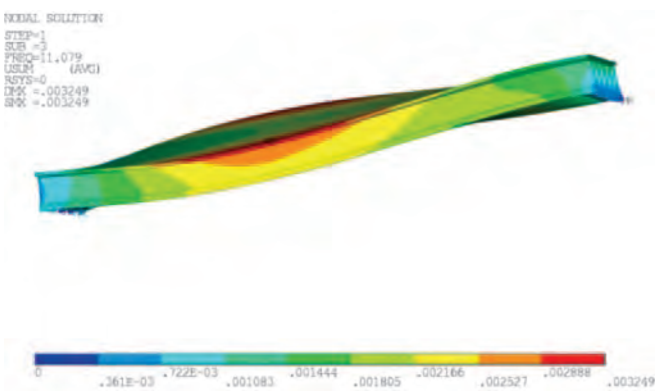

Fig.7 The third order vibration mode

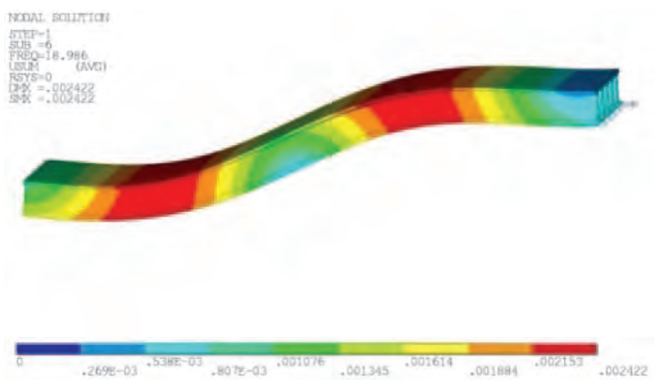

Fig.8 The fourth order vibration mode

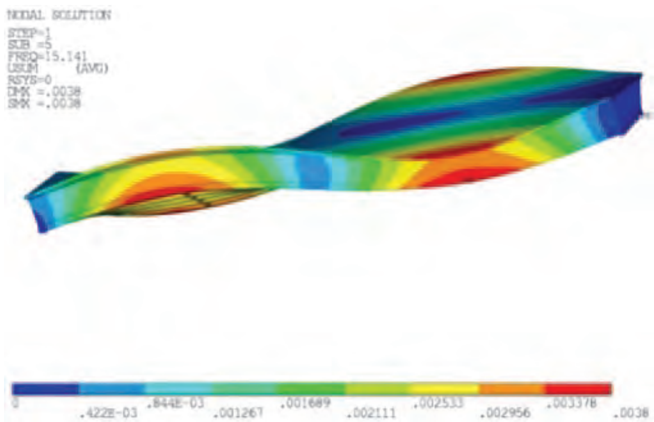

Fig.9 The fifth order vibration mode

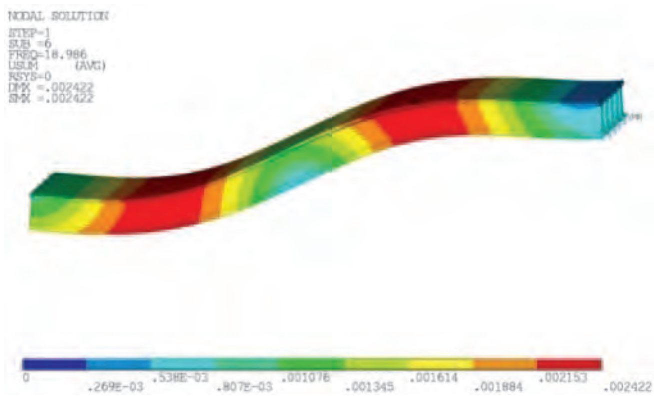

Fig.10 The sixth order vibration mode

small, indicating that it is no relative lateral.The third-order mode is similar to the second-order mode, with torsion occurring and peaks occur in middle of the span. The fourth-order formation is still horizontal vibration, which is the high-order form of the first order, the frequency is increased.The fifth order vibration mode is high-order mode, is the combination of bending and torsion, vibration mode is more complex. Rod also have the corresponding deformation includes stretching, bending, twisting, is a combination of several deformation.

7. Summary

(1) If only load on one lane, the deformation is not uniform, but the deformation is symmetrical when load on double lanes, the size and position of the visible load can affect the deformation and internal force, compared to offset load, although the load size is doubled under full load, the deformation change is not large, so the structure is more benefit under the symmetrical load. The major principal stress of the offset is larger than the full load and the X direction deformation is larger, so it is not well to bear the offset load.

(2) To study resonance phenomena can solve many practical problems. Based on the dynamic performance of the predictable product, it is possible to estimate vibration, noise intensity and other dynamic problems before the product is manufactured and could change the shape of the structure to eliminate or suppress these problems, so the study of the modal is very important.

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