PHYSICS, MATHEMATICS AND PHILOSOPHY†
A Ménage à Trois

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Ménage à Trois, however interesting and exciting it may seem, has serious drawbacks. It is rarely successful or satisfactory to all concerned. The three disciplines : physics, mathematics and philosophy — all including or based on logic — have their separate jurisdictions, aims and methods. While the first two are generally unequivocal and to a great extent uncontroversial in their utterances, philosophers may differ greatly among themselves when discussing the same subject, for instance, about the existence of physical and mathematical entities (ontology) or the manner in which we come to know about the external world (epistemology), with nobody being proved right or wrong in the end. Notwithstanding this wellknown feature about the three enterprises, and the fact that cross-fertilization of ideas from different disciplines can be very fruitful, we should guard against pitfalls when treading border areas. A few examples of such pitfalls and resulting confusion are discussed in this article.

1. Introduction

Physics, mathematics and philosophy, it would appear, all attempt to expand our knowledge of the world we live in. Physics, as a fundamental science, tries to explain, in terms of its concepts and theories, all of inanimate nature. Mathematics acts as an inseparable tool, or rather as a language of physics. Philosophy, among other things,

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examines the nature of knowledge obtained through physics and other sciences.

The jurisdiction, aims and methods of the three disciplines are, we believe, distinct and separate. While physics and mathematics are generally unequivocal and to a large extent uncontroversial in their utterances, philosophers may differ greatly among themselves when discussing the same subject. This is, of course, what is to be expected from the nature of philosophy itself.

Although cross-fertilization of ideas from different disciplines can be very fruitful, one should be careful when crossing borders. Especially when physicists extrapolate results of physics to give answers to ultimate questions, it seems, they frequently commit serious philosophical mistakes. We believe that one may fruitfully combine different sciences for an integrated approach to a problem, but an overall or deeper interpretation of the results of science is a *philosophical activity* and here one needs to exercise extra care. Many popular works on science, it would appear, fail to exercise this care. A few examples of careless "philosophizing" are discussed later (sec 5).

Pursuit of knowledge in science often leads into philosophy. Many philosophers began their careers in physics or mathematics. For success in science, however, one should keep away from metaphysics according to R Courant, a wellknown mathematician. He writes[1] :

'For the scientific procedure it is important to discard elements of metaphysical character and to consider observable facts always as the ultimate source of notions and constructions. To renounce the goal of comprehending "the thing in itself", of knowing the "ultimate truth" of unravelling the innermost essence of the world, may be a psychological hardship for naive enthusiasts, but in fact it was one of the most fruitful turns in modern thinking. Some of the greatest achievements in physics have come as a reward for courageous adherence to the principle of eliminating metaphysics.'

2. The Nature of Physics

Physics may broadly be defined as the study of inanimate nature, of matter and radiation, energy, forces, fields, etc. Modern physics usually starts from describing idealized, simple systems and basic concepts such as mass, velocity, temperature, wave length, electrical charge, fields of force, etc.

According to Bunge[2] the existence of a world in which things move *lawfully,* is a main presupposition of all science. Theories are formed by what is called the *hypothetico deductive method.* (A theory with very wide application is called a Law). A hypothesis is formulated; deductions or consequences of it are tested against observation. A theory must above all stand the test of experiment. Theories are corrigible. Theories are not true or false; they are only more or less successful compared to other theories. When a new theory explains a larger range of observations with a smaller number of basic assumptions (ie, is simpler in a certain sense), explains and predicts data more accurately—it replaces the older theory, as Einstein's Theory of

Gravitation (General Theory of Relativity) replaced Newton's Law.

Theories are formulated almost invariably in terms of mathematical equations where the terms are clearly defined. A set of equations by themselves is never a physical theory; the equations need to be properly interpreted or explained. A theory encompasses a large body of observed data. A main feature of a physical theory is its ability to predict new results. Dirac's relativistic theory of the electron predicted the existence of the positron, and Einstein's General Theory of Relativity predicted the red-shift of spectral lines in the intense gravitational field of the sun[3]. A theory helps us to sort and systematize measured or observed data.

Metaphorically, one might say, theories are like glasses which help us to see the world clearly. All the objects which appear in a physical theory, however, are not directly observable, nor can a theory in most cases be directly verified : only the consequences of a theory are verified by experiment or observation (as in astronomy). According to Bunge[4] all objects in a physical theory, whether observable or not, should show up in some observable effect.

How is the physical world, ie, the world described by the theories of physics, related to our everyday world, the world we perceive? What relation does the physical world have to ultimate reality or the real world out there, especially as theories may change with time? What do we mean by : a theory is or is not a description of reality? These questions do not belong to physics, but rather to philosophy as we understand it.

To recapitulate, the success of physics rests on twin achievements :

* explanation of observation with the help of theories formulated in mathematical terms;

* prediction of results of future experiments or observation.

The theories of physics and the concepts and objects which appear in them, refer to the physical world which is only a model or representation of the real world. Many terms and concepts in a physical theory, however, resemble terms and concepts in our ordinary language describing our everyday world. When dealing with the microworld described by quantum mechanics, or the universe as a whole, we encounter difficulties if the two worlds, the physical and the everyday world, are mixed up. Lastly, the physical theories expressed through mathematics are, strictly speaking, completely formal, even if they contain terms familiar in everyday speech. The theory has to be interpreted, its terms and concepts given a "meaning" before the theory or its consequences can be checked in the world of our experience.

One might here quote Wittgenstein : *'The proposition is a picture of reality. The proposition is a model of the reality as we think it is.'*

3. What is Mathematics about

The famous mathematician G H Hardy[5] talks of mathematics as dull and useful on the one hand and useless and wonderful on the other. He talks exuberantly of *'the real mathematics of real mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann.'*

The subject matter of mathematics, it would appear, are not physical objects, nor are empirical observations, the ultimate grounds for deciding the truth or falsity of a mathematical statement. Mathematical knowledge seems to be a case of pure rational knowledge, gained by thinking alone and independent of empirical verification. We might just list the different viewpoints on mathematics :

Platonism	— Mathematical objects are abstract entities which exist independently of our thoughts;
Formalism (Hilbert)	— Manipulation of undefined symbols according to certain basic rules;
Logicism (Frege, Russel)	— Arithmetic is a part of logic; arithmetic was reduced to set theory, but this, it seems, cannot be genuinely regarded as part of logic;
Intuitionism (Brouwer)	— Mathematics is something directly apprehended and therefore capable of being constructed.

For a beginner, in search of certainty, objective knowledge that is eternally valid, mathematics might appear as infallible and able to provide the right answer. However this hope might prove to be a mirage on a closer examination. We might recall that in physics too we encountered a similar situation : a physical theory describes a model, an abstraction of some slice of reality, not the reality itself.

An interesting paper by Hao Wang presented in Kirchberg in August 1991[6] discusses the views of Gödel and Wittgenstein on the nature of mathematics and on the nature of philosophy.

Both Gödel and Wittgenstein considered philosophy to be concerned with the fundamental. Gödel held that in mathematics we discover rather than invent or create. Wang seems to support this. Wittgenstein held an opposite view. In general, Gödel valued the abstract more than the concrete in philosophy, in constrast to Wittgenstein.

In his article, Hao Wang points out that the two philosophers differed from each other in their conceptions of truth and proof in mathematics, of relation of science and philosophy, value of metaphysics and the importance of language in philosophy. *'Wittgenstein and Gödel agree that conceptual investigations, as the characteristic task of philosophy, are to see clearly what we already have.'* They also emphasize "absolute clarity" and on being "unprejudiced". Gödel seemed, like Huesserl and unlike Wittgenstein, to set the goal of *'philosophy* *as rigorous science.'* Wittgenstein's opposition to set theory is well known. *('The theory of classes (sets) is completely superfluous in Mathematics.')* Hao Wang in this paper also gives arguments for his own preferences. For instance, Wang finds : we can make mathematical concepts precise (Gödel), also, we can't make philosophical concepts precise (Wittgenstein). Wang says, the infinite is the central issue in studying the foundations of mathematics. As a contrast, he quotes Wittgenstein as saying : *'Infinity is not so mysterious as it looks; how very misleading the expressions of Cantor are.'*

The purpose of referring to Wang's article was to point out that although mathematics as the queen of sciences is an exact science par excellence, when one talks about the nature of mathematics, there can be large differences and disagreements among different thinkers.

4. Philosophy, A Big question Mark

It is well known that, what philosophy is, is in itself a philosophical question with no agreed answer. We confine ourselves to what might be called analytical philosophy, whose aim is conceptual analysis[7]. Here even alternative viewpoints are there proposed by different philosophers and one can decide for oneself which one appears most satisfactory and reasonable.

In his search for certainty, of the kind one expects in logic and science, Bertrand Russel thought philosophy should emulate science, philosophy would then be just another science, more general and abstract than other sciences, but equally well-grounded on logic. Science and philosophy are akin in method and product according to Russel[8].

Wittgenstein, from his early career on, held a different view. He said already in his *Notes on Logic* that *'philosophy is wholly distinct from science'*. Later he says in *Tractatus* : *'The object of philosophy is the logical clarification of thoughts. Philosophy is not one of the natural sciences. The word philosophy must mean something which stands above or below, but not beside the natural sciences. Elsewhere he describes, philosophical activity as a striving for a deeper understanding of language. The sciences are totally different in nature and pursued for very different purposes. Consequently, science is irrelevant for philosophy[8].'*

In his post-1929 works, Wittgenstein wrote more deeply upon the nature of philosophy than any other major philosopher since Kant according to Baker and Hacker[8]. We can only refer to this masterly work and other works for a proper presentation of Wittgenstein's conception of philosophy. Suffice it to say that Wittgenstein's opposition to the scientific conception of philosophy seems to remain constant.

There are bound to be different opinions about what Wittgenstein said on tasks of philosophy. For the purpose of this essay, however, notwithstanding what his critics say, Wittgenstein serves as an excellent example of one trying to illuminate the border area between science and philosophy. In the next section, a few concrete cases of physics and mathematics will be discussed which have often been given a distorted interpretation by popular writers, or scholars writing in a popular style.

5. Dangers of Careless Border Crossing

We have selected four examples of modern scientific and mathematical topics which have generated a lot of interest also among the general reader because of their supposed significance to our everyday life. As already mentioned one should—we believe—use extra care when dealing with such problems for popular consumption. One should, for the sake of honesty and clarity, state clearly where physics and mathematics end and where philosophy or speculation starts. There is, of course, no reason to place science or philosophy, one above the other, but just to remember their different natures. A motivation for writing this article has been the frequent exaggeration noticed in popular writings by scientists on these topics.

We might say right away, that quantum and relativity theories have very little impact in our day-to-day lives where we deal with a range of magnitudes, covered under what is called meso-physics. We should also note that formal results of mathematics can be extrapolated to social systems only very roughly.

5.1 Heisenberg's Inequalities

These relations, introduced by Heisenberg in 1925, constitute a central theorem of quantum mechanics. They are also called the Uncertainty Principle of Heisenberg. According to it, one cannot know, even in principle, simultaneously the position q and momentum p (mass x velocity) of an electron. The product of the uncertainties of q and p is of the order of h, the Planck's constant ($h = 6.62559 \times 10^{-34}$ JS) :

$$\Delta p \times \Delta q \geq h.$$

Stated differently, the electron does not have a precise position and a precise momentum at the same time.

This is a rather simplified statement of the principle, but the main point is that its interpretation has caused a lot of controversy among scientists and philosophers of science for over 60 years. Are the Heisenberg's relations — uncertainties (our ignorance) or indeterminacies (objective quantum mechanical indeterminacies) or something else?

According to Mario Bunge[9] they are neither uncertainties nor indeterminacies. *These mathematical relations follow from basic postulates of quantum mechanics.* They should be understood as saying that at the quantum level (for microobjects like electrons) the mean standard deviations of, say, the position and momentum of an electron are inversely related so that if one is large the other is small. Subjective uncertainties do not have a place in physical theories according to Bunge and others. Bunge says[9] : *'both the uncertainty and the indeterminacy interpretations rest on the tacit hypothesis that microobjects are point-like.*

But this hypothesis does not belong to quantum mechanics.' If an electron is point-like, then of course it has a definite position at all times even if we are unable to compute or measure it for some reason or other. Bunge suggests that electrons are neither (classical) particles nor (classical) waves, they are objects of a kind unknown to classical physics. They might be called *quantons*, for instance. They are extended objects with no definite shape or boundary. They have no sharp position. Normally the quanton has a nonvanishing spatial halfwidth and a nonvanishing momentum halfwidth. This explanation by Bunge allows us to view the most famous inequality in modern physics in a new light without astonishment (how is it possible that a particle cannot have a precise position and momentum at the same time?).

The interpretation of quantum mechanics has raised a lot of questions right from the beginning. Various "philosophies" have been offered by scientists beginning with Bohr, Heisenberg, Einstein but the difficulties of "interpretation" of quantum mechanics seem to persist. A way out of these difficulties, we suggest, may be found if we remember that in philosophy there is always bound to be alternative points of view (with different degrees of plausibility).

Interpretation of quantum mechanics comes under the domain of philosophy and one should therefore not be surprised if rival interpretations do not agree. One can choose an interpretation that one finds convincing; *none is true or false*. We find the interpretation given by Bunge[4] preferable to the standard or orthodox Copenhagen Interpretation of quantum mechanics.

The edifice of modern physics is built upon a set of interconnected theories expressed in mathematical terms—they are to a large extent successful in explaining and predicting observations. Research in theoretical physics consists in advancing this success still farther, *not in approaching closer to reality*.

5.2 Space-time and the beginning of the world

While the concepts of space and time in physics, astronomy and cosmology may not pose any mystery to a pure scientist (who does not cross border zones into philosophy), they continue to fill our minds with a sense of wonder and mystery beyond comprehension.

In the study of space and time, we see that physics and mathematics have had an excellent marriage indeed. Euclid's 3-dimensional geometry corresponds to our perception of space. Ideas of absolute space and a continuous flow of time (strictly speaking, we do not experience time, but only events in time) appeared unproblemetical to a common man.

These ideas also served as a basis of physics until the advent of modern physics in the twentieth century.

With the epoch-making discoveries of Gauss, Riemann, Minkowski, Lorentz, Einstein, Eddington, Stephen Hawking and other cosmologists, the present picture of the world conceived in physics shows a vast difference from the one experienced by us. Four dimensional space-time, curved space near matter as an explanation of gravitation, expanding universe (expanding into what ?), the origin and evolution of the universe starting from a singularity, a Big Bang that took place 15 billion years ago, (what was there before the bang?) — all appear a bit puzzling to put it mildly. We can neither form any picture in our minds, nor find any parallel in our world of experience to what the scientists say.

What do the physicists and astronomers say? In brief, they postulate theories and compare observations with the consequences of their theories. To relate a theory to our world of experience or to explain the universe as a whole may be quite problematical. It is essentially a philosophical task and we should not expect a definitive answer on questions such as when did the world or time begin, or what are the boundaries of space. Choosing Immanuel Kant as an outstanding philosopher, let us see what he says.

Kant says that both space and time, besides a few other concepts such as causality, are inborn categories of human thought, built-in into our thinking process. We cannot think of anything without using the ideas of space and time. Similarly cause and effect are inseparably connected in whatever we perceive.

Kant further says that we cannot have a definite knowledge about supersensible matters, eg, the world in its unconditional totality. In his famous antinomy of pure reason, Kant argues that both the opposing statements —

* the world must have a beginning in time, and
* the world has no beginning in time

— can be proved to be true. Thus the question, *when did the world begin*, must remain unanswerable. The question, it would appear, is a philosophical one, rather than scientific. In that case Kant's response may be one of the best ones we have in hand. We might here quote from a commentator of Kant[10] :

'Suppose we were to accept the Big Bang hypothesis concerning the origin of the universe. Only a short-sighted person would think that we have then answered the question how the world began. For what caused the bang? Any answer will suppose that something already existed. So the hypothesis cannot explain the origin of things. The quest for an origin leads us forever backwards in the past. But either it is unsatisfiable, —in which case how does cosmology explain the existence of the world? — or it comes to rest in the postulation of a causa sui — in which case we have left the scientific question unanswered and taken refuge in theology. Science itself pushes us towards the antinomy, by forcing us always to the limits of nature.'

Bunge[2] criticizes the doctrine of creationism which proposes creation of everything from nothing. He suggests that the only thing we can say is that the universe began a new phase in its evolution at the time of the supposed Big Bang; this explosive event was probably preceded by a stage in which matter was in some respects different from what we know today.

5.3 Chaos theory

Chaos in a system can be defined as random fluctuations that are deterministic in origin. Nonlinearity is a necessary but not a sufficient condition for chaos. The chaos theory deals with the solution of nonlinear differential equations, where—for some values of parameters in the equations—random or chaotic behaviour ensues even if completely deterministic laws are followed by the system. Chaos theory is often classed with the relativity and quantum theories as the most outstanding discovery in physics in this century[11].

Using a slightly technical language, let us consider a dissipative nonlinear system whose motion may be described by a trajectory in phase space. Depending on the value of the control parameter, the trajectory will tend, with the passage of time, towards one of the following alternatives :

A point attractor—a stable final state;

A periodic attractor—a closed curve in phase space;

A quasi-periodic attractor—nonrepetitive periodic motion;

Chaotic, strange or fractal attractor, eg, Lorenz attractor.

[By attractor we mean a region in the phase diagram to which a point representing the motion of a system, is attracted.]

The actual source of chaos is the property of the nonlinear system of separating initially close trajectories exponentially fast in a bounded phase space.

The predictions of chaos theory have found corroboration in different physical sciences: physics, chemistry, hydrodynamics, meteorology, animal populations, etc. The numerical constants in the equations controlling the onset of chaos, was found to be the same in widely different types of systems, showing a universality of the phenomena. This is again an example of a good marriage between physics and mathematics.

Chaos theory gained popularity partly because of the term chaos, which is frequently used in daily conversation. One notices frequently the tendency to apply mathematical models to sociological problems, often forgetting that social systems by their very nature cannot be expected to behave like a wellcontrolled physical experiment. There are too many unknown, unquantifiable parameters in a social system, such as, a group of people, a nation, etc.

When chaos theory is applied to social cases, one should particularly bear this in mind and not make any predictions about the onset of disorder, riots, stock market crashes with the same confidence as one might show in the case of a pendulum or a water pipe. Already in the case of weather prediction, one cannot predict more than a few days in advance because of the underlying non-linear equations of meteorology susceptible to chaotic evolution. [cf, E N Lorenz : *Does the flap of a butterfly wing in Brazil set off a tornado in Texas?* 1979]

5.4 Gödel's Incompleteness Theorems

Kurt Gödel (1906-1978) was one of the greatest mathematicians and logicians of this century. His work, popularly entitled the Incompleteness Theorem (1931), was one of the most significant discoveries in the field of mathematics[12]. This, however, does not justify presenting Gödel's results as implying that no knowledge can ever be complete.

Human knowledge has been known to be incomplete from the earliest days of philosophy. What Gödel showed was applicable, although in a overwhelming way, only in a particular branch of mathematics, eg, the formal system of arithmetic[13].

Gödel's (first) Incompleteness Theorem shattered the formalist hopes of David Hilbert (1862-1943) who thought that all possible mathematical truths could be captured within some formal system. Gödel astonished the mathematical and philosophical world by proving that for any formal system F that is

* finitely describable,
* consistent, and
* strong enough to prove the basic facts about elementary arithmetic,

i) F is incomplete, and

ii) F cannot prove its own consistency.

There are several other popular ways of expressing Gödel's Incompleteness Theorem. We might quote a couple of them for the interested readers :

> All consistent axiomatic formulations of number theory include undecidable propositions.

> In any formal system F of arithmetic, there will be a sentence P of the language of F such that if F is consistent, neither P nor its negation can be proved in F.

[Gödel's work has been the starting point of a lot of deep philosophizing, eg, clarifying our notions of truth, proposition and proof.]

6. Conclusion

We have seen that physics, as the most basic and fundamental of the exact sciences, cannot do without mathematics. In addition to mathematics, physics needs concepts and a model to work with. Above all, physics needs theories to explain observed phenomena and predict new results.

Physical theories describe, in mathematical terms, what we might call the physical world containing physical entities. This physical world is supposed to be a model, constructed by human thought of some part of reality. In order to verify a physical theory, its results and their consequences are translated back to our everyday world in which experiments and observations are made. Theories are corrigible. They are neither true nor false, but more or less successful in explaining and predicting.

Successful theories of the microworld (quantum mechanics) or of the universe as a whole (cosmology), often cause difficulties of understanding in terms of what we can visualize and perceive in our everyday world for mainly two reasons.

Firstly we forget the difference between the two worlds : physical and the everyday world; secondly, language contributes to confusion. Terms and concepts in the two worlds, because of similarity of expressions, get mixed up.

Philosophy as a discipline, we consider, to be different from the sciences. Interpretation or explanation of physical phenomena beyond a certain preliminary level, we think, is a philosophical activity. In science, scientists tend towards accepting unanimously the best available theory, whereas in philosophy it may be difficult to choose between rival positions. Writers on popular science would do well to remember the difference between science and philosophy and guard against unjustified generalizations.

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